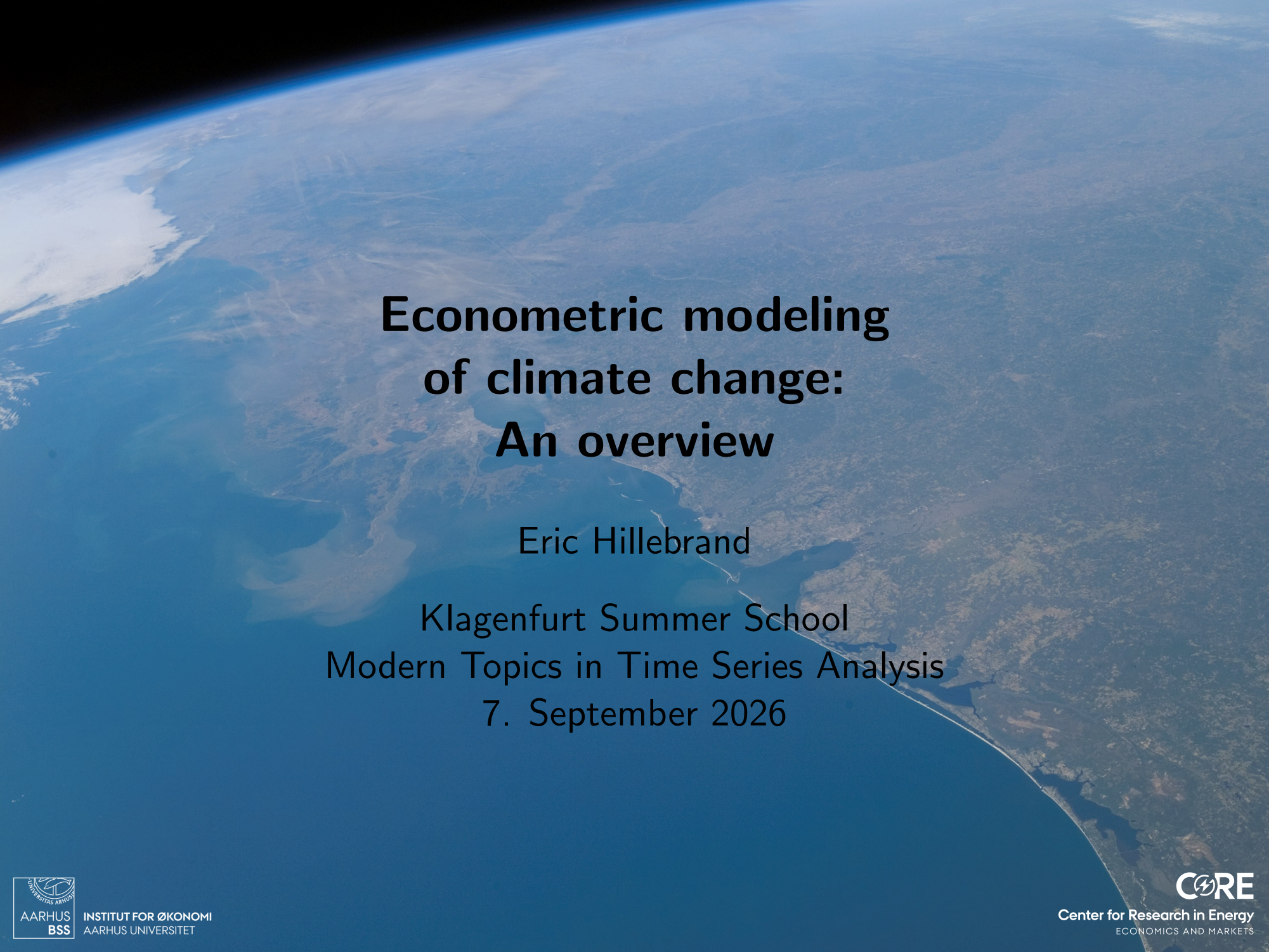


Econometric modeling of climate change: An overview

Eric Hillebrand

Klagenfurt Summer School
Modern Topics in Time Series Analysis
7. September 2026

AARHUS BSS
INSTITUT FOR ØKONOMI
AARHUS UNIVERSITET

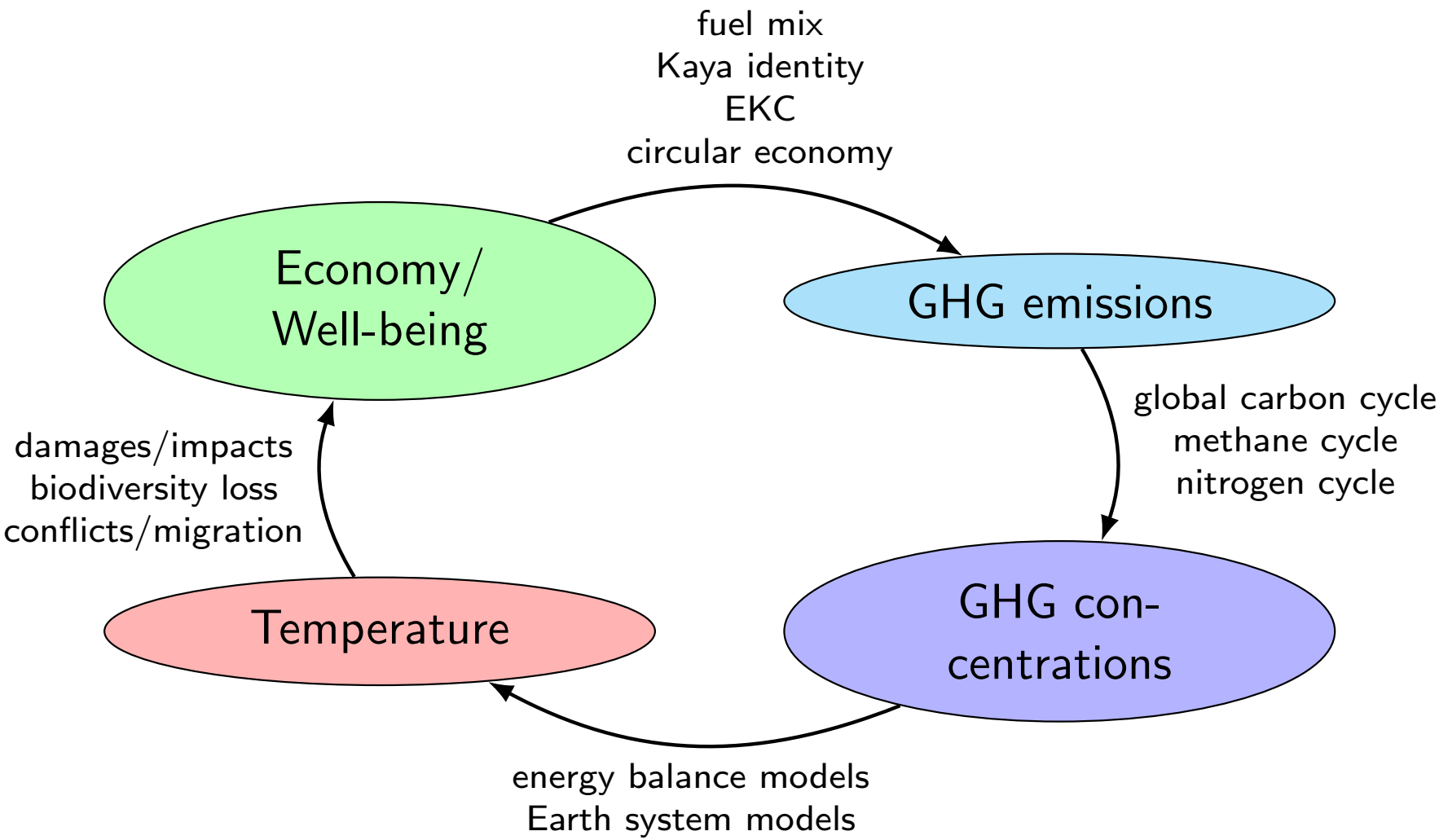
CORE
Center for Research in Energy
ECONOMICS AND MARKETS

Outline

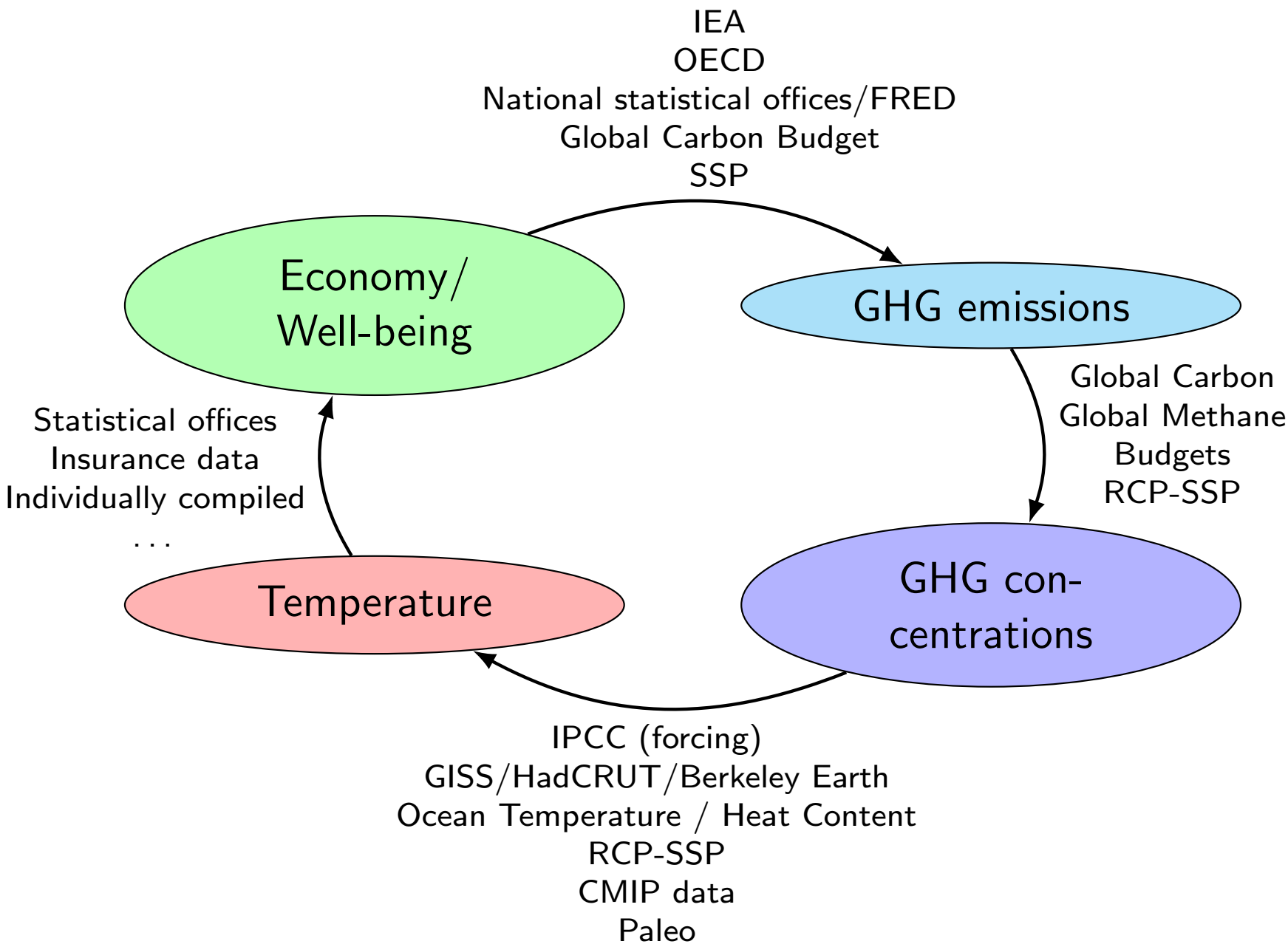
1. Climate econometrics: A selected overview
   - 1.1 Impacts of temperature rise
   - 1.2 CO_2 atmospheric concentrations to temperatures
   - 1.3 CO_2 emissions to concentrations
   - 1.4 Macroeconomic activity to CO_2 emissions
2. The amazing linearity
3. The CO_2–climate record in deep time

1. Climate econometrics: A selected overview

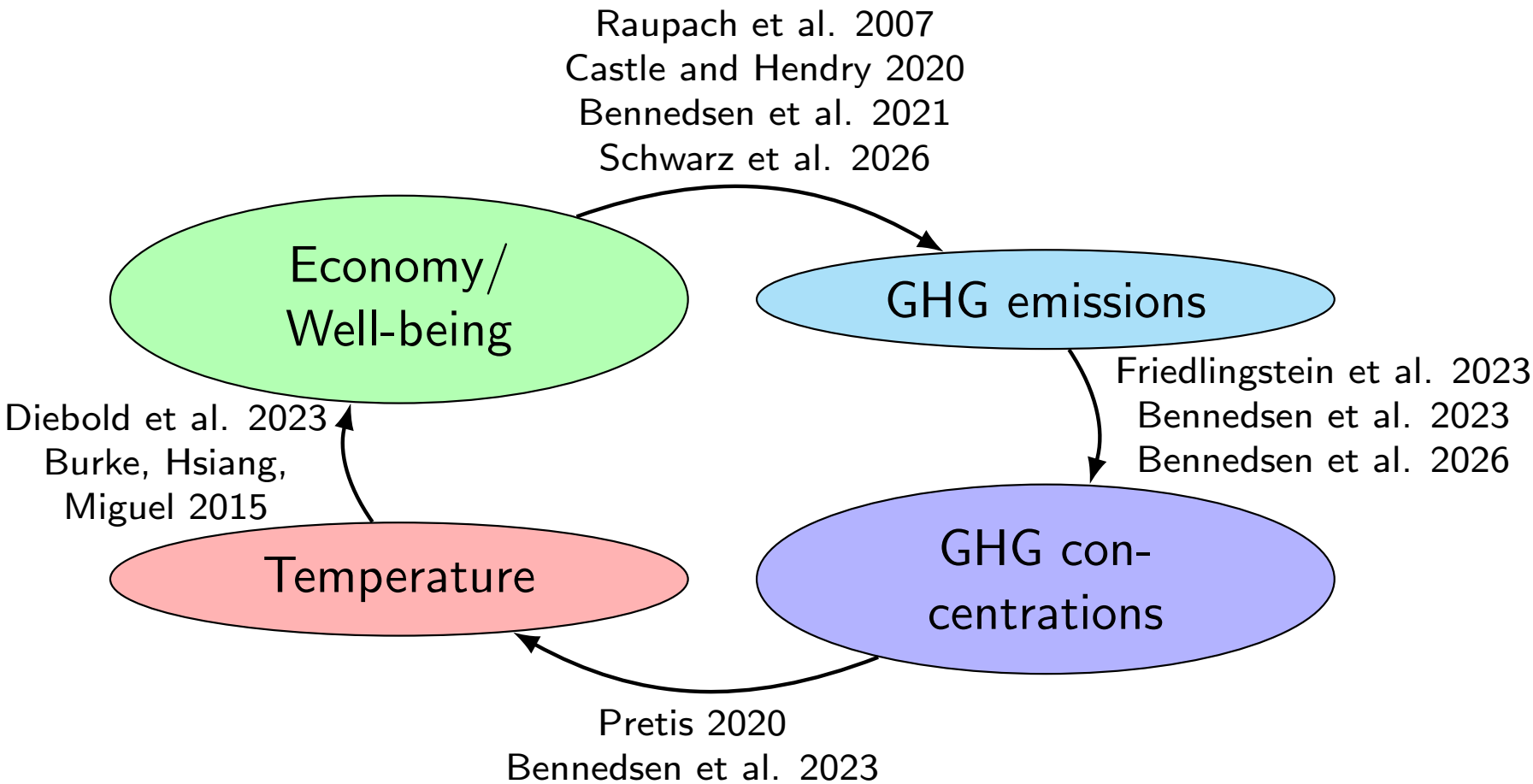
Climate change and economy interaction



Data


IEA
OECD
National statistical offices/FRED
Global Carbon Budget
SSP
Economy/
Well-being
GHG emissions
Global Carbon
Global Methane
Budgets
RCP-SSP
Statistical offices
Insurance data
Individually compiled
...
Temperature
GHG con-
centrations
IPCC (forcing)
GISS/HadCRUT/Berkeley Earth
Ocean Temperature / Heat Content
RCP-SSP
CMIP data
Paleo

Case studies



Impacts (Temperature $\rightarrow x$)

Sea ice

Diebold, Rudebusch, Göbel, Coulombe, Zhang (2023), When will Arctic sea ice disappear? Projections of area, extent, thickness, and volume, *Journal of Econometrics 236(2), 105479*

Sea ice area and extent: Let ΔS_i denote the surface area of a grid cell in the Arctic ocean, and $c_{i,t}$ the ice concentration measured by satellite.

$$SIA_t = \sum_i c_{i,t} \mathbb{1}_{c_{i,t} \geq 0.15} \Delta S_i, \qquad SIE_t = \sum_i \mathbb{1}_{c_{i,t} \geq 0.15} \Delta S_i$$

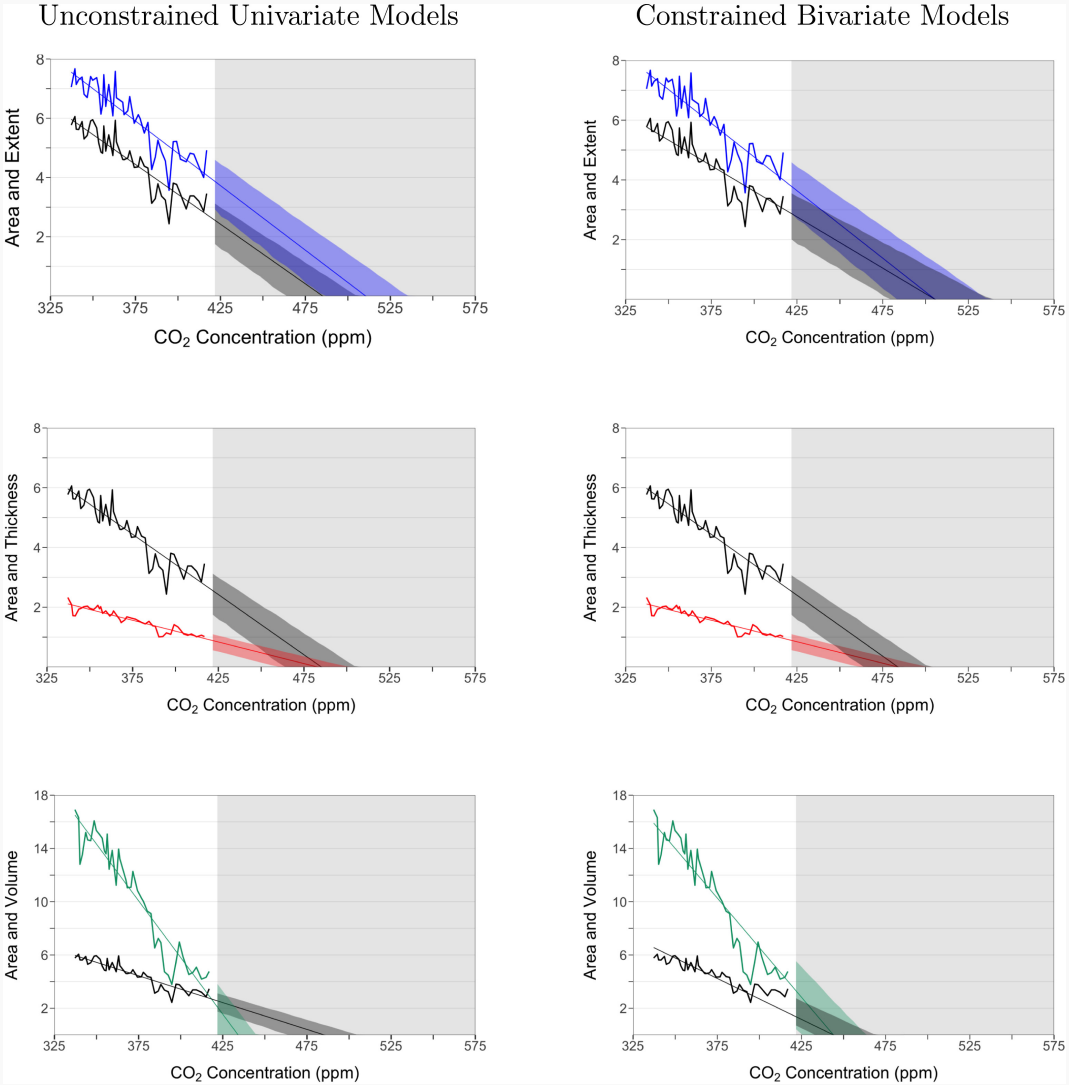
Model: Let C_t be atmospheric CO_2 concentrations.

$$SIA_t = a_1 + b_1 C_t + \varepsilon_{1,t},$$
$$SIE_t = a_2 + b_2 C_t + \varepsilon_{2,t}.$$

Restrict such that $SIA_t = SIE_t = 0$ for the same level of atmospheric concentrations: $b_2 = \frac{a_2 b_1}{a_1}$.

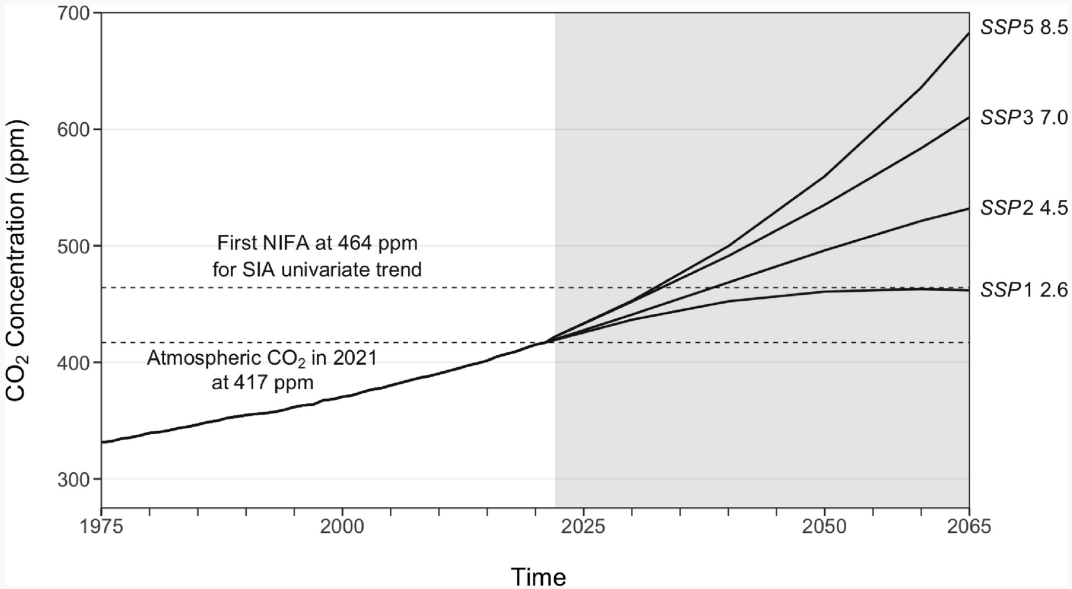
Impacts (Temperature $\rightarrow x$)

Sea ice



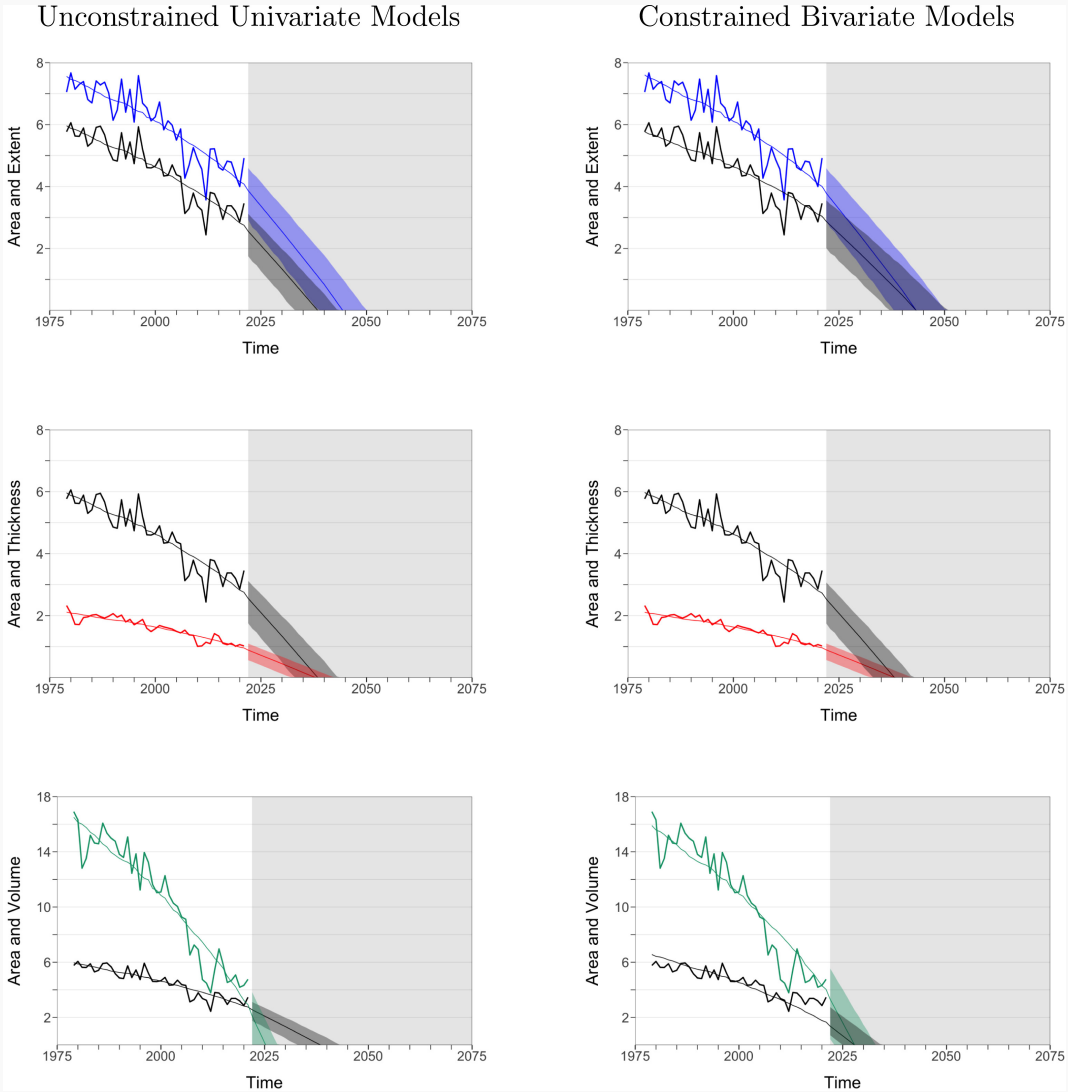
Impacts (Temperature $\rightarrow x$)

Sea ice



Impacts (Temperature $\rightarrow x$)

Sea ice



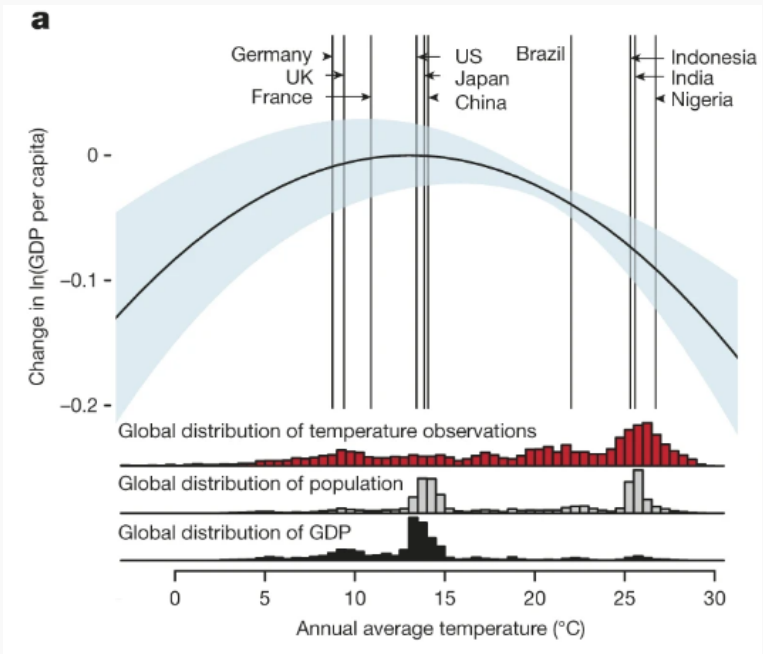
Impacts (Temperature $\rightarrow x$)

GDP

Burke, Hsiang, Miguel (2015), Global non-linear effect of temperature on economic production, *Nature* 527, 235–239.

Model: Let $\Delta Y_{i,t}$ be log differences in GDP of country i at time t, $T_{i,t}$ temperature, $P_{i,t}$ precipitation.

$$\Delta Y_{i,t} = \mu_i + \nu_t + \theta_{i,1} t + \theta_{i,2} t^2 + \beta_1 T_{i,t} + \beta_2 T_{i,t}^2 + \lambda_1 P_{i,t} + \lambda_2 P_{i,t}^2 + \varepsilon_{i,t}$$



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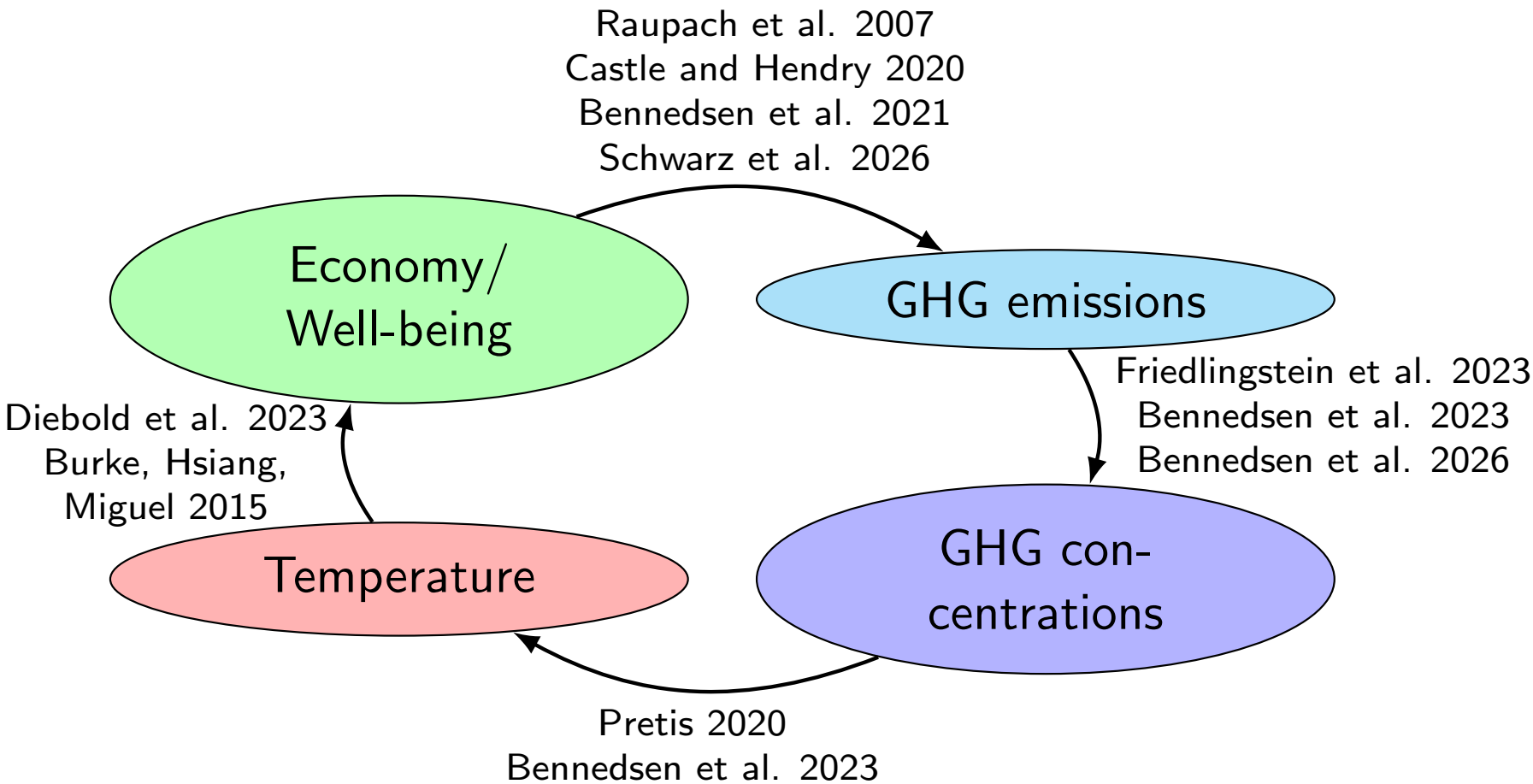
Raupach et al. 2007
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Temperature
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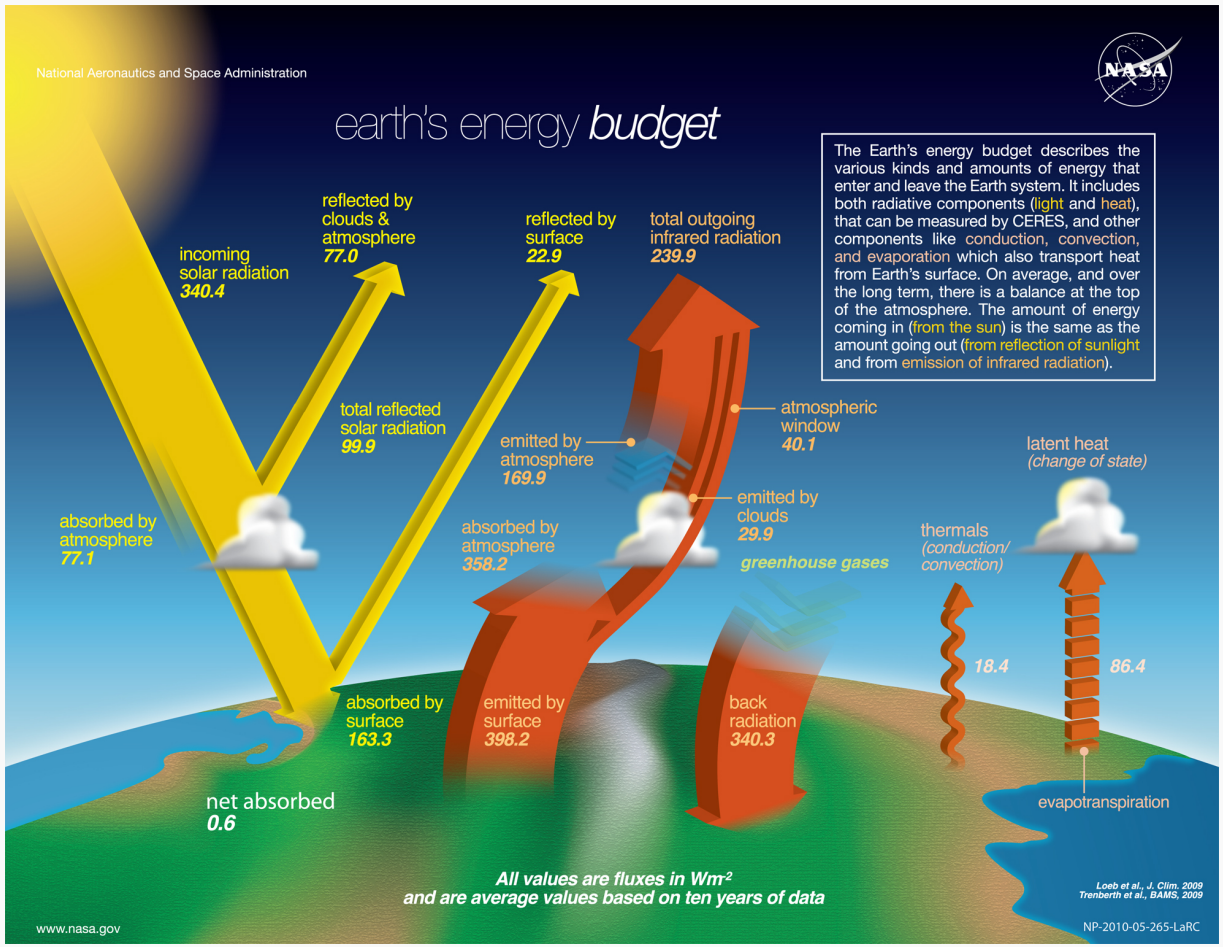
Pretis 2020
Bennedsen et al. 2023

GHG concentrations → Temperature

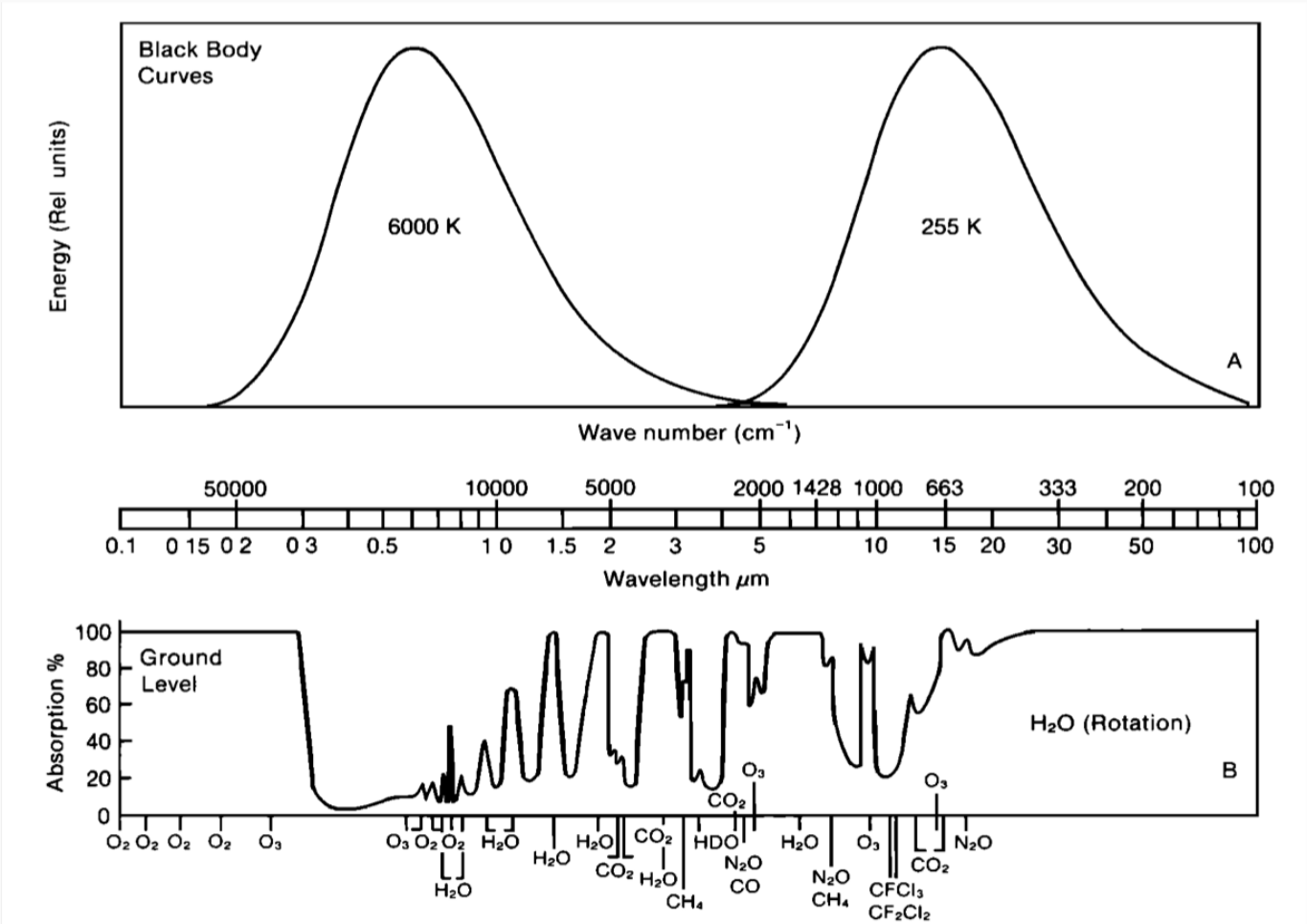
Energy balance



Source: Wikipedia

GHG concentrations → Temperature

Energy balance



Source: Goody and Yung (1989), Atmospheric Radiation: Theoretical Basis, 2nd ed., OUP

GHG concentrations $\rightarrow$ Temperature

EBM as CVAR

Pretis (2020), Econometric modelling of climate systems: The equivalence of energy balance models and cointegrated vector autoregressions, *Journal of Econometrics* 214, 256–273.

A two-layer energy balance model can be written as follows. Let m denote a surface and "middle" layer, d a "deep" layer. Let C_m be the heat capacity of the upper ocean, C_d the heat capacity of the deep ocean. Let T_m be the average temperature of the ocean surface and middle layer. Let T_d be the temperature of the deep layer. Let F be the radiative forcing, both natural (solar and volcanic) and anthropogenic (greenhouse gases and aerosols).

$$C_m \frac{dT_m}{dt} = F - \lambda T_m - \gamma(T_m - T_d),$$
$$C_d \frac{dT_d}{dt} = \gamma(T_m - T_d).$$

GHG concentrations $\rightarrow$ Temperature

EBM as CVAR

Discretize in an Euler approximation:

$$\Delta T_{m,t} = \frac{1}{C_m}[-\lambda T_{m,t-1} + F_{t-1}] - \frac{\gamma}{C_m}[T_{m,t-1} - T_{d,t-1}] + u_{1,t},$$
$$\Delta T_{d,t} = \frac{\gamma}{C_d}[T_{m,t-1} - T_{d,t-1}] + u_{2,t}.$$

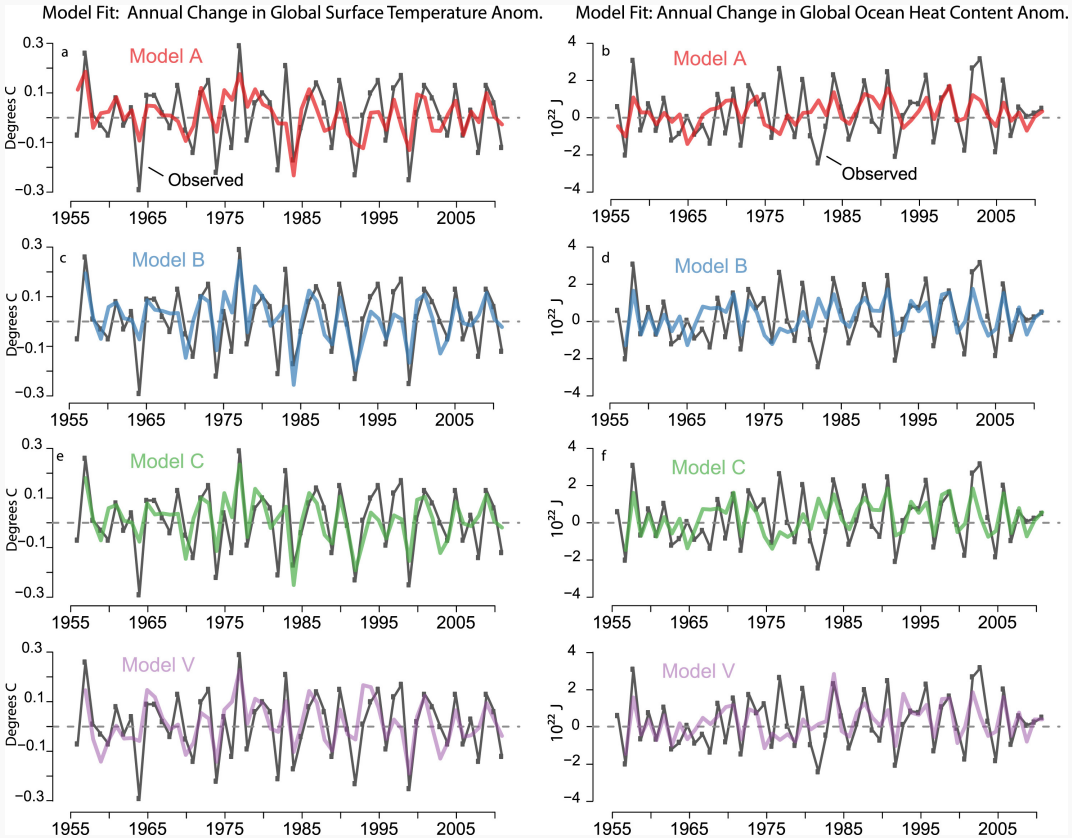
This is a cointegrated vector autoregression (CVAR) of first order with error-correction representation

$$\Delta X_t = \alpha \beta^T X_{t-1} + u_t$$

$$\begin{bmatrix} \Delta F_t \\ \Delta T_{m,t} \\ \Delta T_{d,t} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{1}{C_m} & -\frac{\gamma}{C_m} \\ 0 & \frac{\gamma}{C_d} \end{bmatrix} \begin{bmatrix} 1 & -\lambda & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} F_{t-1} \\ T_{m,t-1} \\ T_{d,t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ u_{1,t} \\ u_{2,t} \end{bmatrix}$$

GHG concentrations → Temperature

EBM as CVAR



GHG concentrations $\rightarrow$ Temperature

EBM as state space model

Bennedsen, Hillebrand, Zhou Lykke (2023), Global temperature projections from a statistical energy balance model using multiple sources of historical data, *Journal of Climate* 36(19), 6817–6838

Same framework:

$$C_m \frac{dT_m}{dt} = F - \lambda T_m - \gamma(T_m - T_d),$$
$$C_d \frac{dT_d}{dt} = \gamma(T_m - T_d).$$

Divide $F_t = N_t + A_t$ into natural and anthropogenic forcing and model the anthropogenic part

$$A_t = \beta_t + A_{t-1} + \eta_{A,t-1},$$
$$\beta_t = \beta_{t-1} + \eta_{\beta,t-1}$$

GHG concentrations $\rightarrow$ Temperature

EBM as state space model

Write as a linear state space model (Durbin and Koopman (2012), Time Series Analysis by State Space Methods):

$$\begin{aligned} \text{Measurement equation } y_t &= Z\alpha_t + \varepsilon_t, & \varepsilon_t &\sim \mathrm{N}(0, H), \\ \text{State equation } \alpha_{t+1} &= T\alpha_t + \eta_t, & \eta_t &\sim \mathrm{N}(0, Q). \end{aligned}$$

State equation: $\alpha_t = (T_{m,t}, T_{d,t}, A_t, \beta_t)^T$

$$\begin{aligned} T_{m,t} &= \left[1 - \frac{\lambda + \gamma}{C_m}\right] T_{m,t-1} + \frac{\gamma}{C_m} T_{d,t-1} + \frac{1}{C_m}(N_{t-1} + A_{t-1}) + \eta_{1,t-1}, \\ T_{d,t} &= \frac{\gamma}{C_d} T_{m,t-1} + \left[1 - \frac{\gamma}{C_d}\right] T_{d,t-1} + \eta_{2,t-1}, \\ A_t &= \beta_{t-1} + A_{t-1} + \eta_{3,t-1}, \\ \beta_t &= \beta_{t-1} + \eta_{4,t-1}. \end{aligned}$$

$$Q \in \mathbb{R}^{4\times 4}.$$

GHG concentrations $\rightarrow$ Temperature

EBM as state space model

Measurement equation: $y_t = (\{Y_{T_m,i,t}\}_{i=1,\dots,I}, \{Y_{T_d,j,t}\}_{j=1,\dots,J}, \{Y_{O,j,t}\}_{j=1,\dots,J}, Y_{F,t})^T$

$$
\begin{aligned}
Y_{T_m,i,t} &= T_{m,t} + \varepsilon_{1,i,t}, && i = 1, \dots, I, \\
Y_{T_d,j,t} &= T_{d,t} + \varepsilon_{2,j,t}, && j = 1, \dots, J, \\
Y_{O,j,t} &= C_d T_{d,t} + \varepsilon_{3,j,t}, && j = 1, \dots, J, \\
Y_{F,t} &= N_t + A_t + \varepsilon_{4,t}
\end{aligned}
$$

$H \in \mathbb{R}^{(I+2J+1)\times(I+2J+1)}$ diagonal.

GHG concentrations $\rightarrow$ Temperature

EBM as state space model

TABLE 2. Summary of the temperature anomaly, OHC anomaly, and forcing datasets employed in the empirical analysis. The last column "baseline" indicates the reference period or the year upon which the anomalies are constructed.

Variable	Acronym/type	Institution/authors	Reference	Coverage	Baseline
GMST anomalies	GISTEMP	NASA	GISTEMP Team (2021) Lenssen et al. (2019)	1880–2020	1951–80
	NOAAGlobalTemp	NOAA	Menne et al. (2018)	1880–2019	1901–2000
	HadCRUT5	Met Office Hadley Centre	Morice et al. (2021)	1850–2020	1961–90
	BEST	Berkeley Earth	Rohde and Hausfather (2020)	1850–2020	1951–80
	CW14	Cowtan and Way (2014)	Cowtan and Way (2014)	1850–2020	1961–90
	JMA	Japanese Meteorological Agency	Japan Meteorological Agency (2021)	1891–2020	1981–2010
Global ocean temperature anomalies	NOAA yearly, 0–700 m	NOAA	Levitus et al. (2012)	1955–2020	Not specified
	IAP yearly 0–700 m	Institute of Atmospheric Physics	Cheng et al. (2017)	1940–2020	1981–2010
	IAP yearly 0–2000 m	Institute of Atmospheric Physics	Cheng et al. (2017)	1940–2020	1981–2010
Global OHC anomalies	NOAA yearly, 0–700 m	NOAA	Levitus et al. (2012)	1955–2020	Not specified
	IAP yearly 0–700 m	Institute of Atmospheric Physics	Cheng et al. (2017)	1940–2020	1981–2010
	IAP yearly 0–2000 m	Institute of Atmospheric Physics	Cheng et al. (2017)	1940–2020	1981–2010
	Yearly 0–700 m, 0–2000 m	Zanna et al. (2019)	Zanna et al. (2019)	1870–2018	1870
Forcing		IPCC	Smith et al. (2021a)	1750–2019	1750

GHG concentrations → Temperature

EBM as state space model

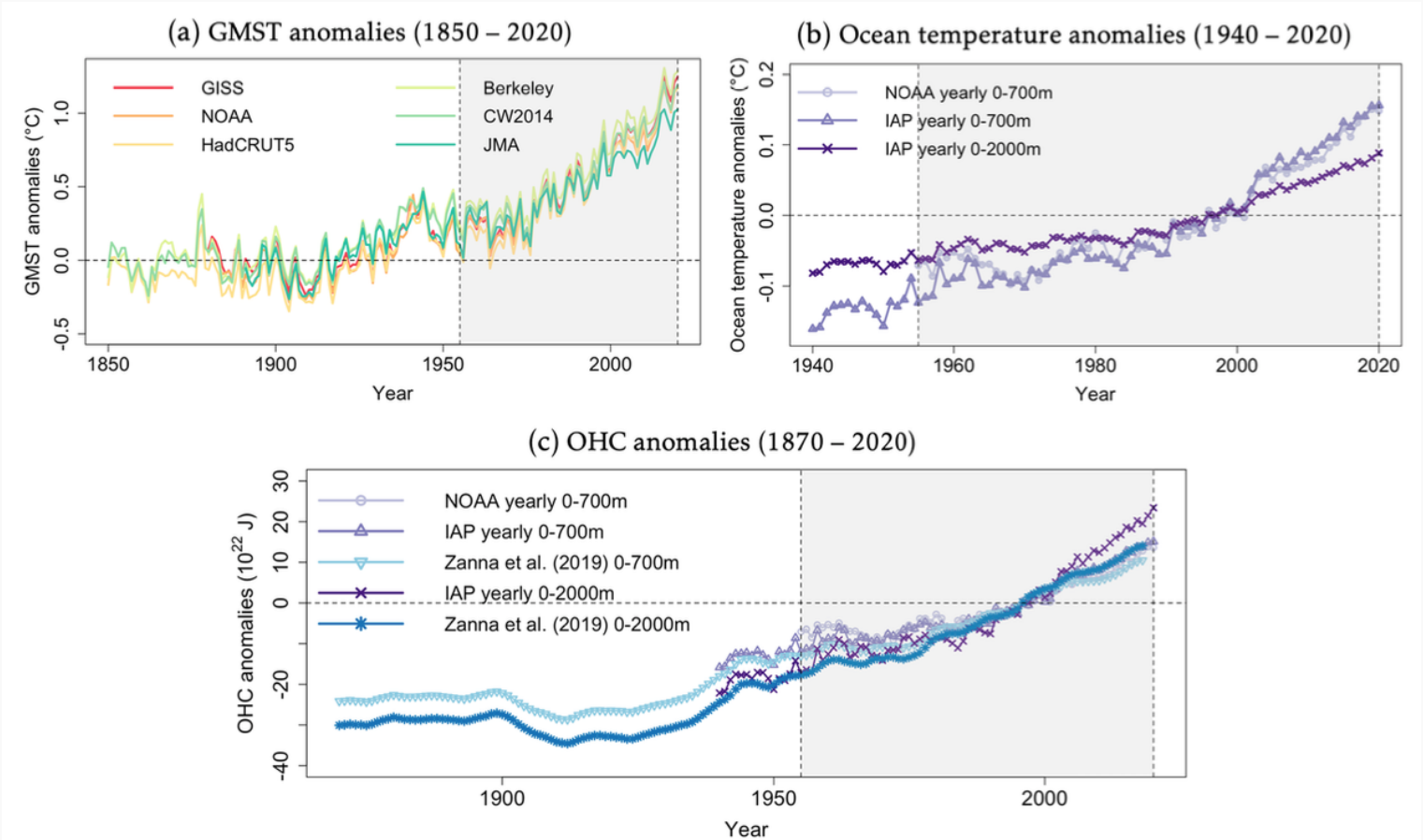
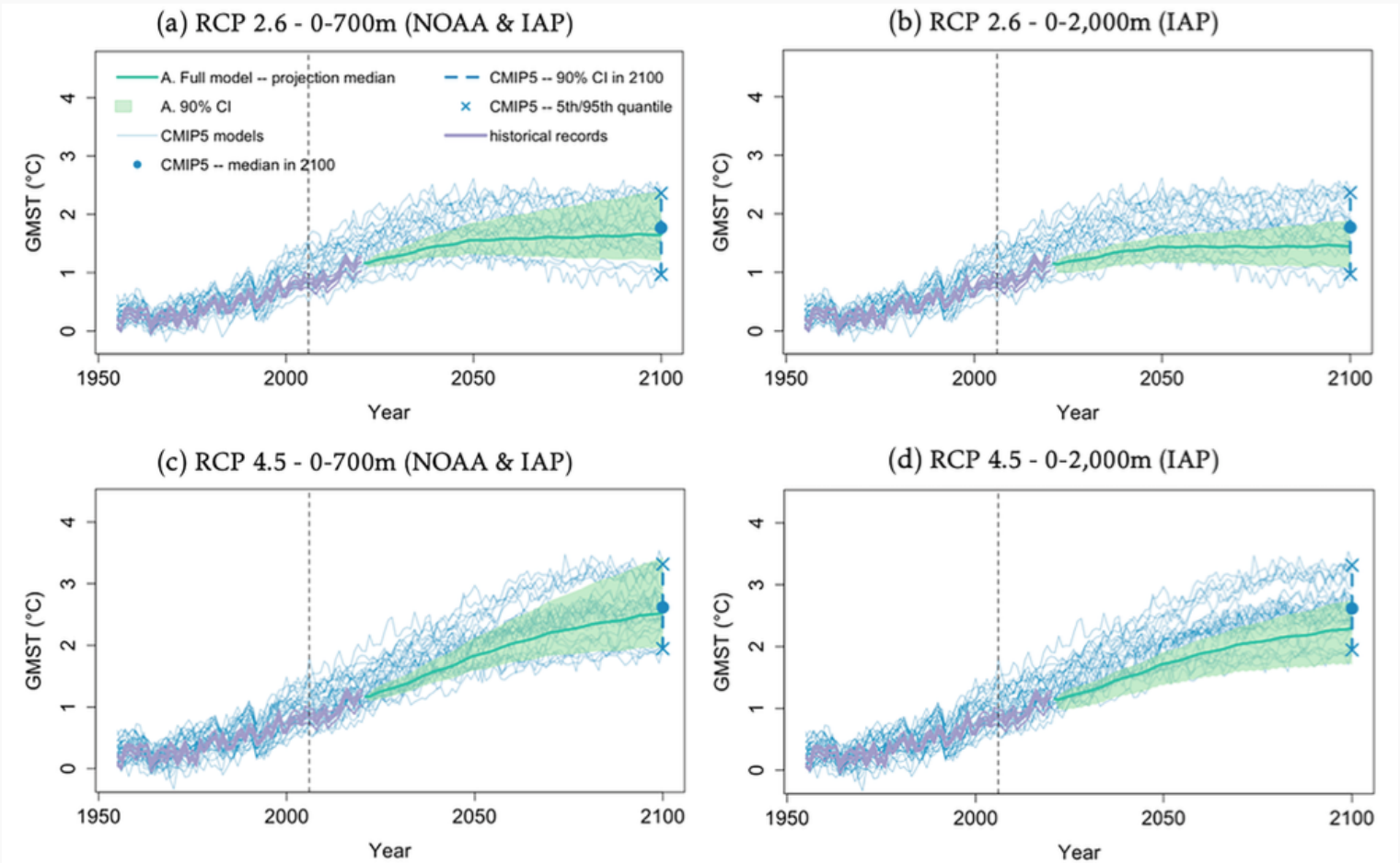


FIG. 4. (a) GMST, (b) ocean temperature, and (c) OHC anomaly series after synchronization. GMST anomalies are synchronized with respect to the preindustrial era of 1850–1900, while ocean temperature and OHC anomalies are synchronized with respect to 1981–2010. The shaded area corresponds to the sample period 1955–2020 in the empirical study.

GHG concentrations $\rightarrow$ Temperature

EBM as state space model



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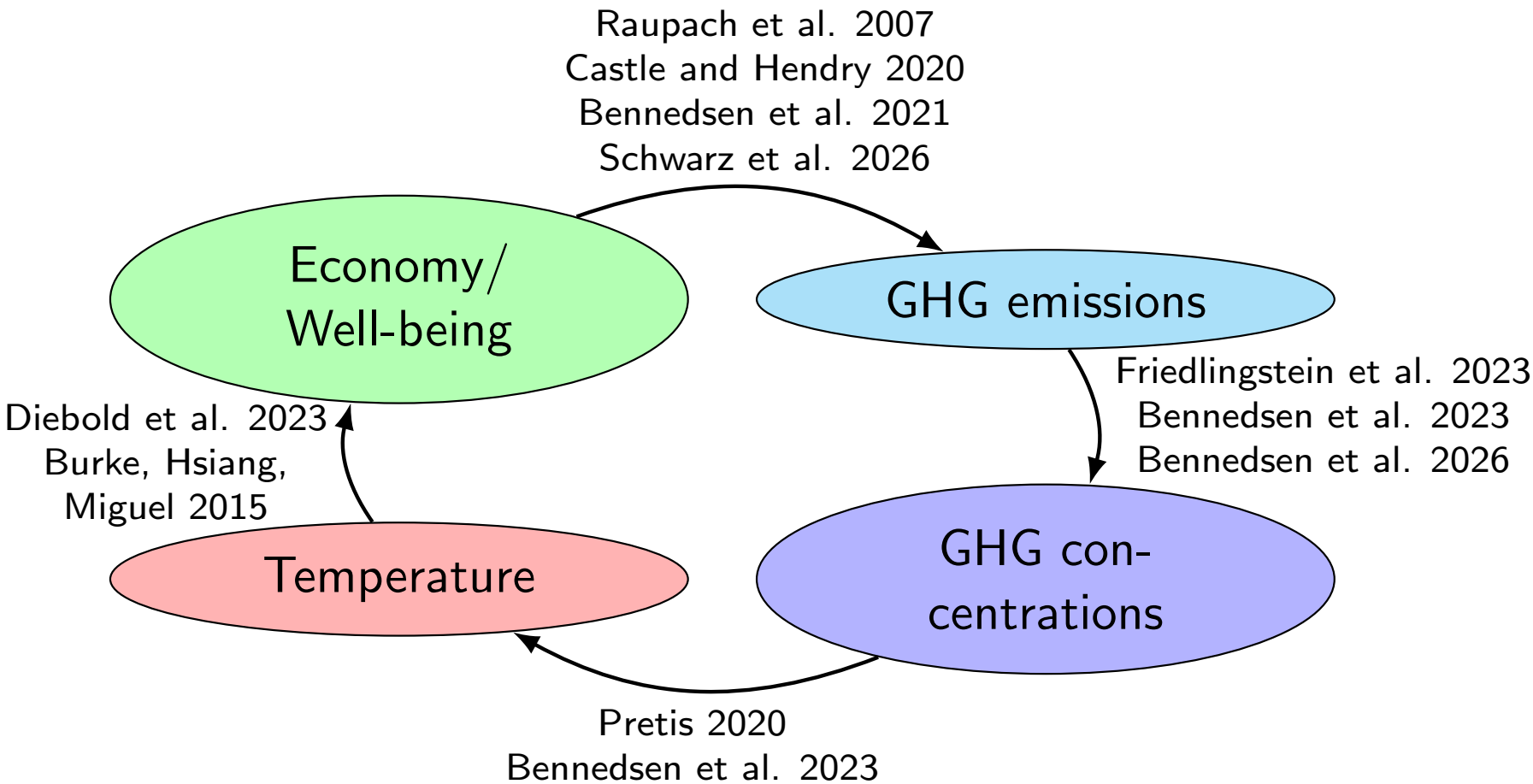
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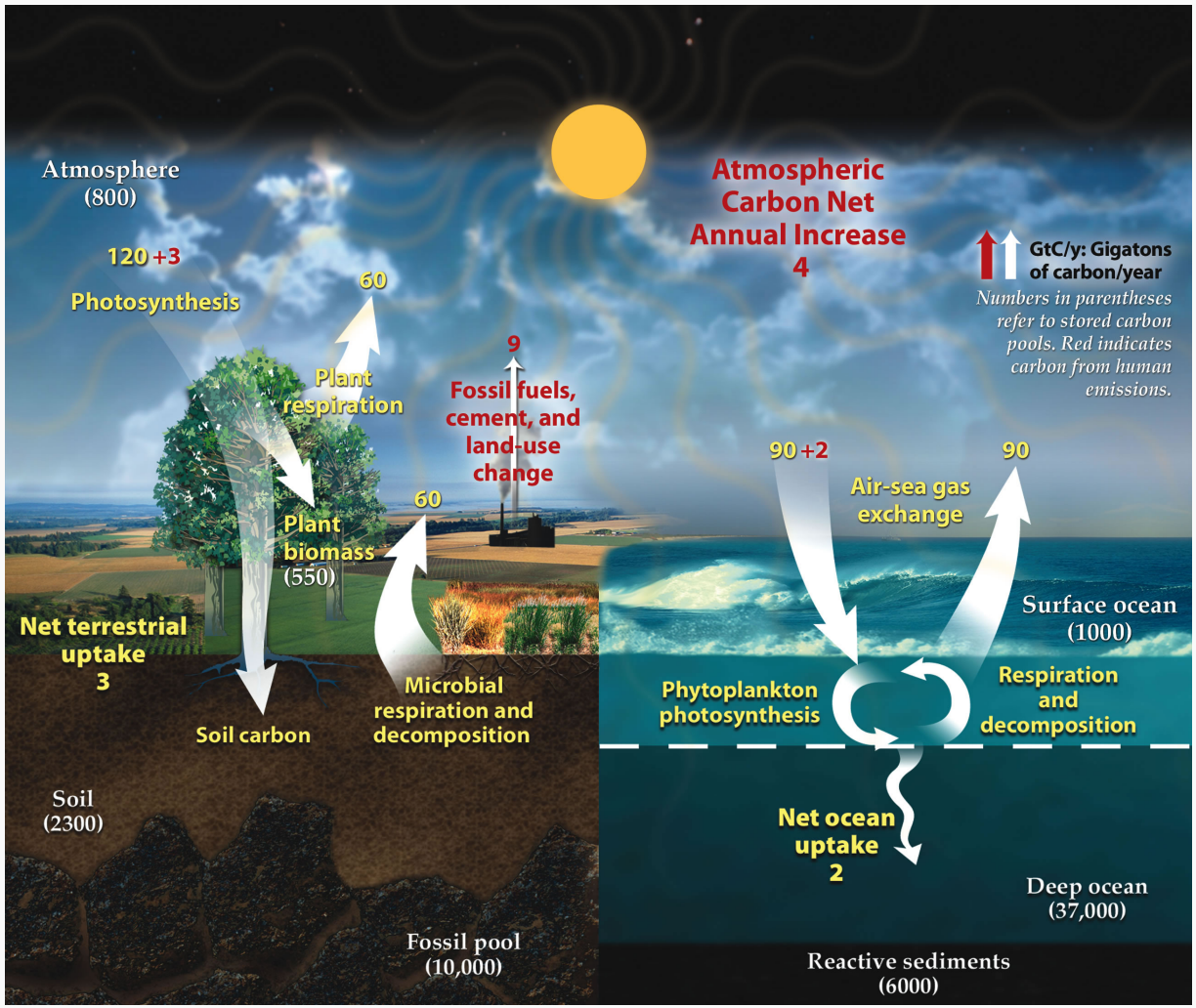
GHG emissions $\rightarrow$ atmospheric GHG concentrations

Global carbon cycle

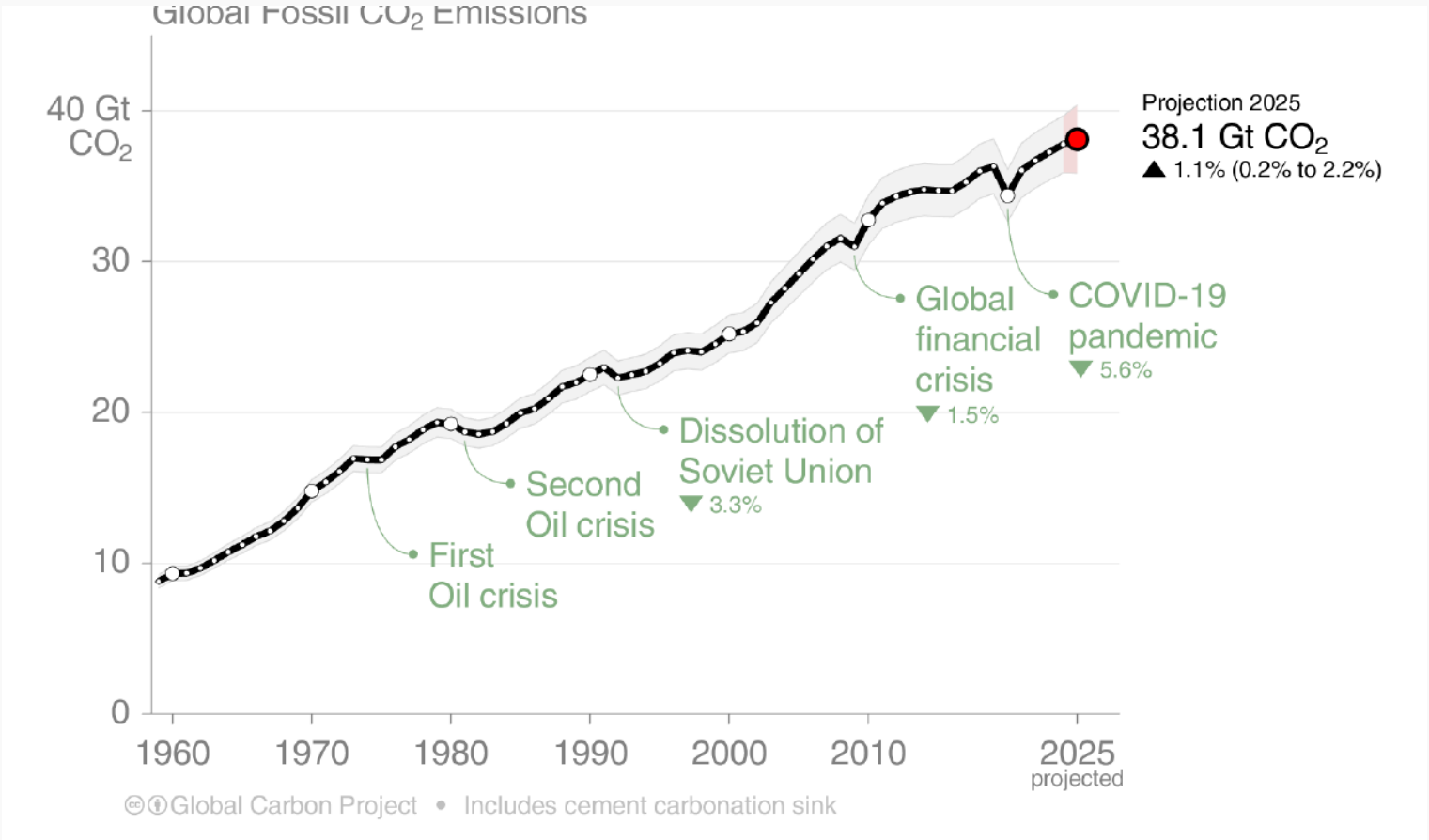


Source: Wikipedia

GHG emissions $\rightarrow$ atmospheric GHG concentrations

Global carbon budget

Friedlingstein et $\sim$120 al. (2025), Global Carbon Budget 2025, *Earth System Science Data* 18, 3211–3288.



GHG emissions $\rightarrow$ atmospheric GHG concentrations

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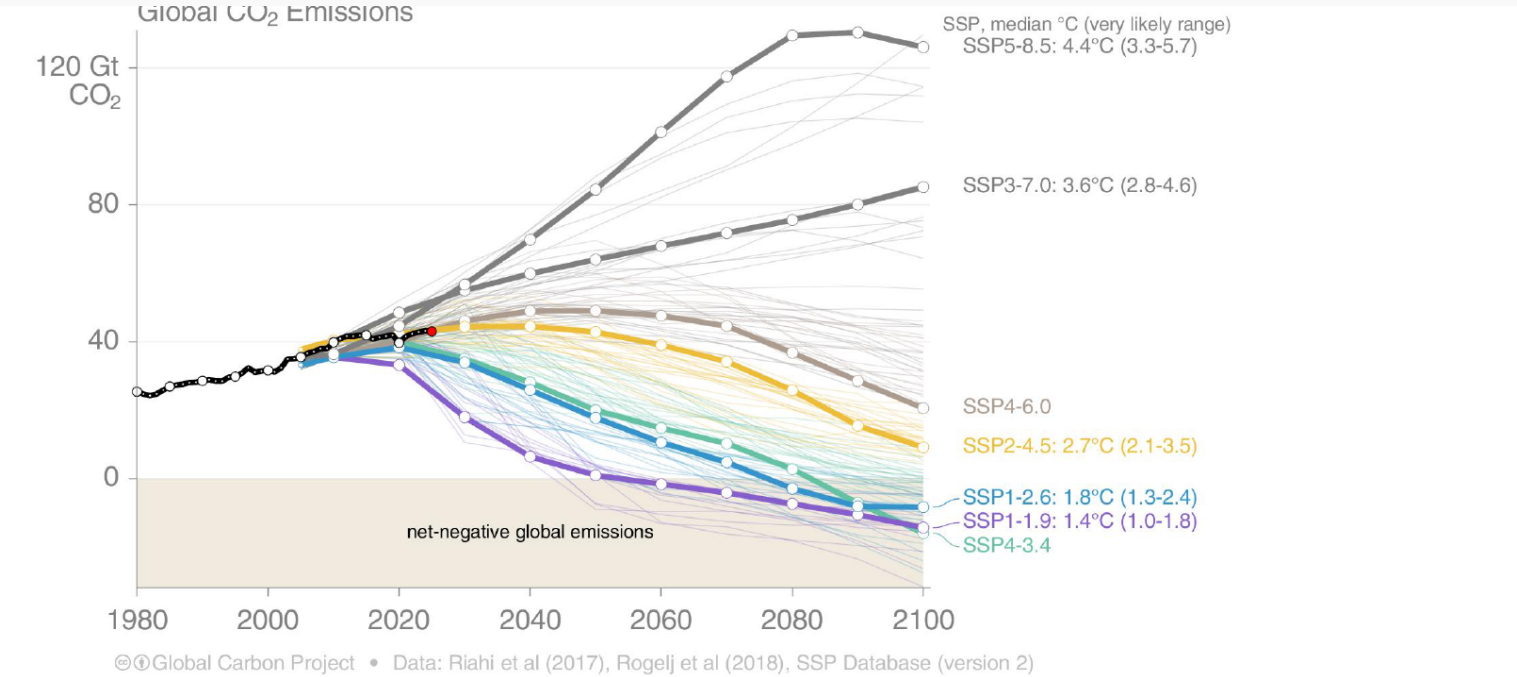

GHG emissions $\rightarrow$ atmospheric GHG concentrations

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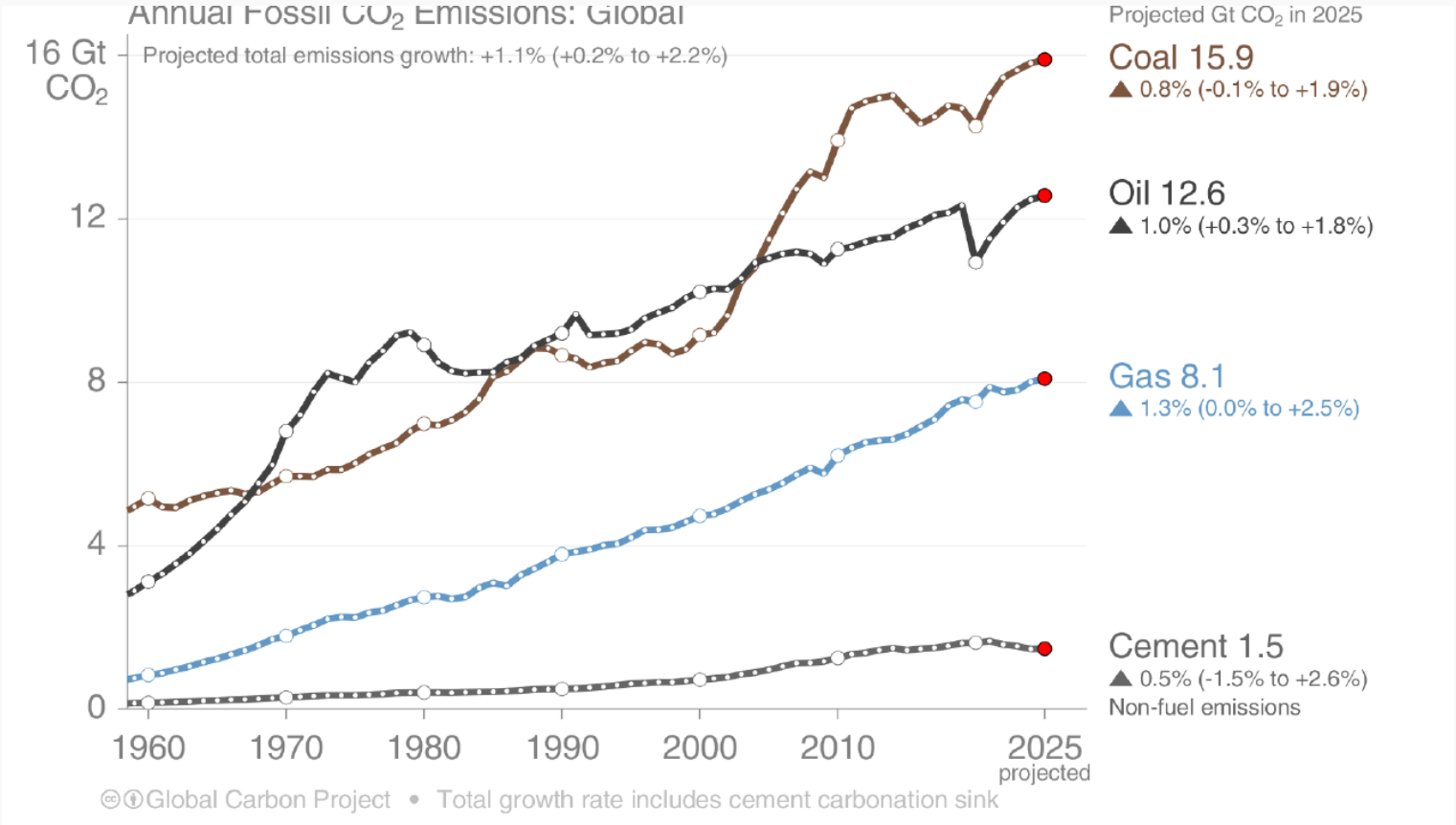

GHG emissions $\rightarrow$ atmospheric GHG concentrations

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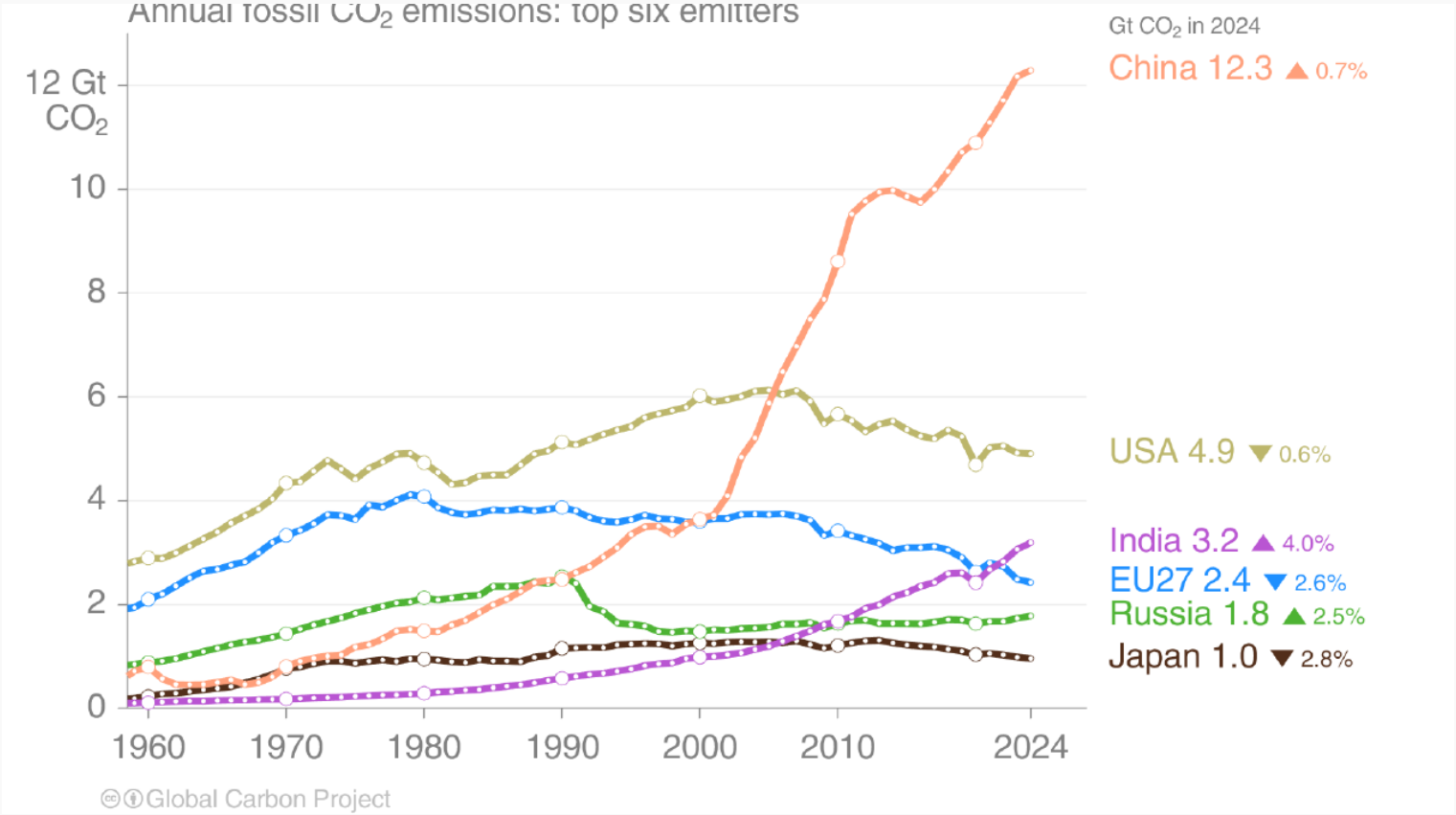

GHG emissions $\rightarrow$ atmospheric GHG concentrations

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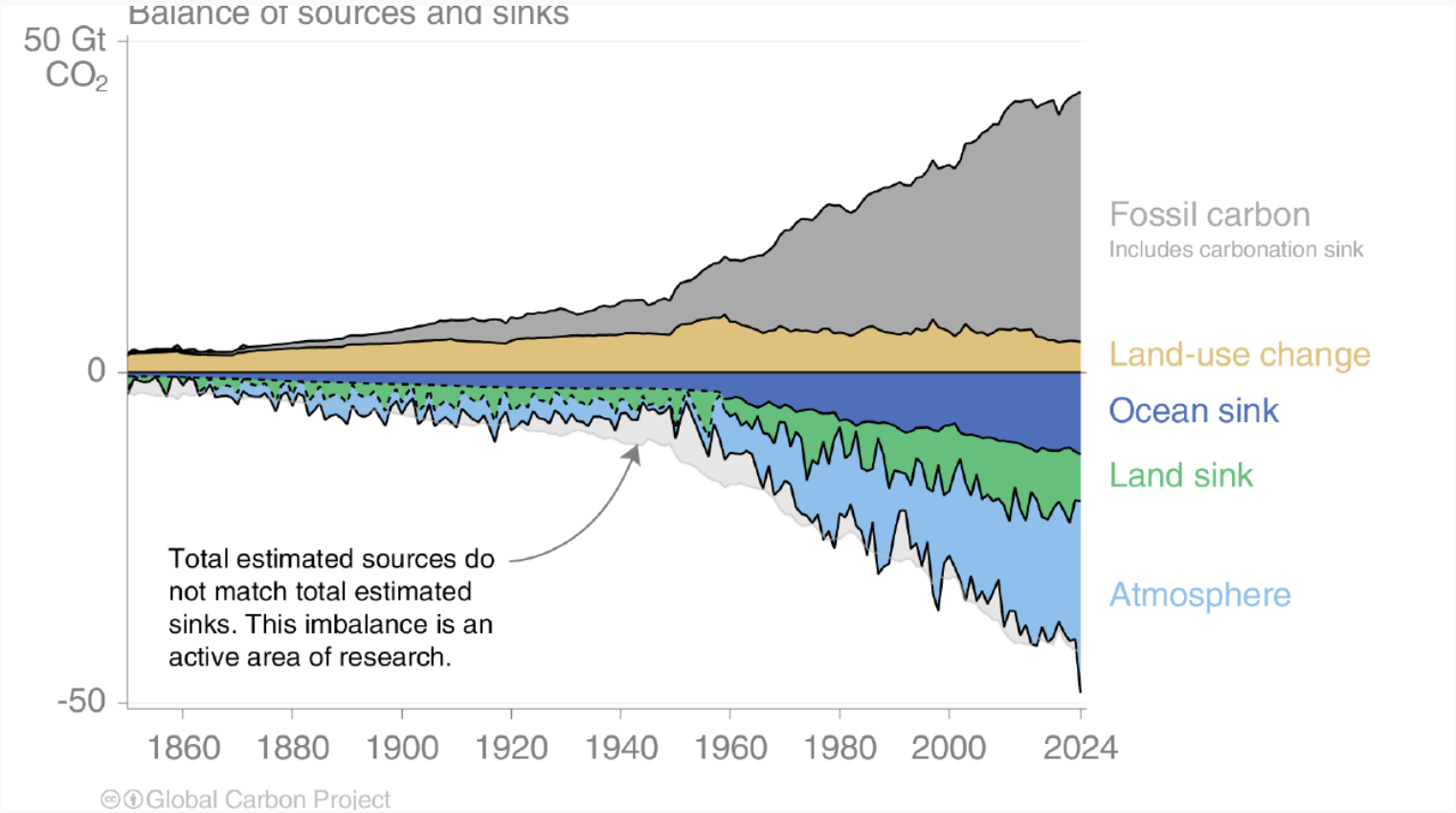

GHG emissions $\rightarrow$ atmospheric GHG concentrations

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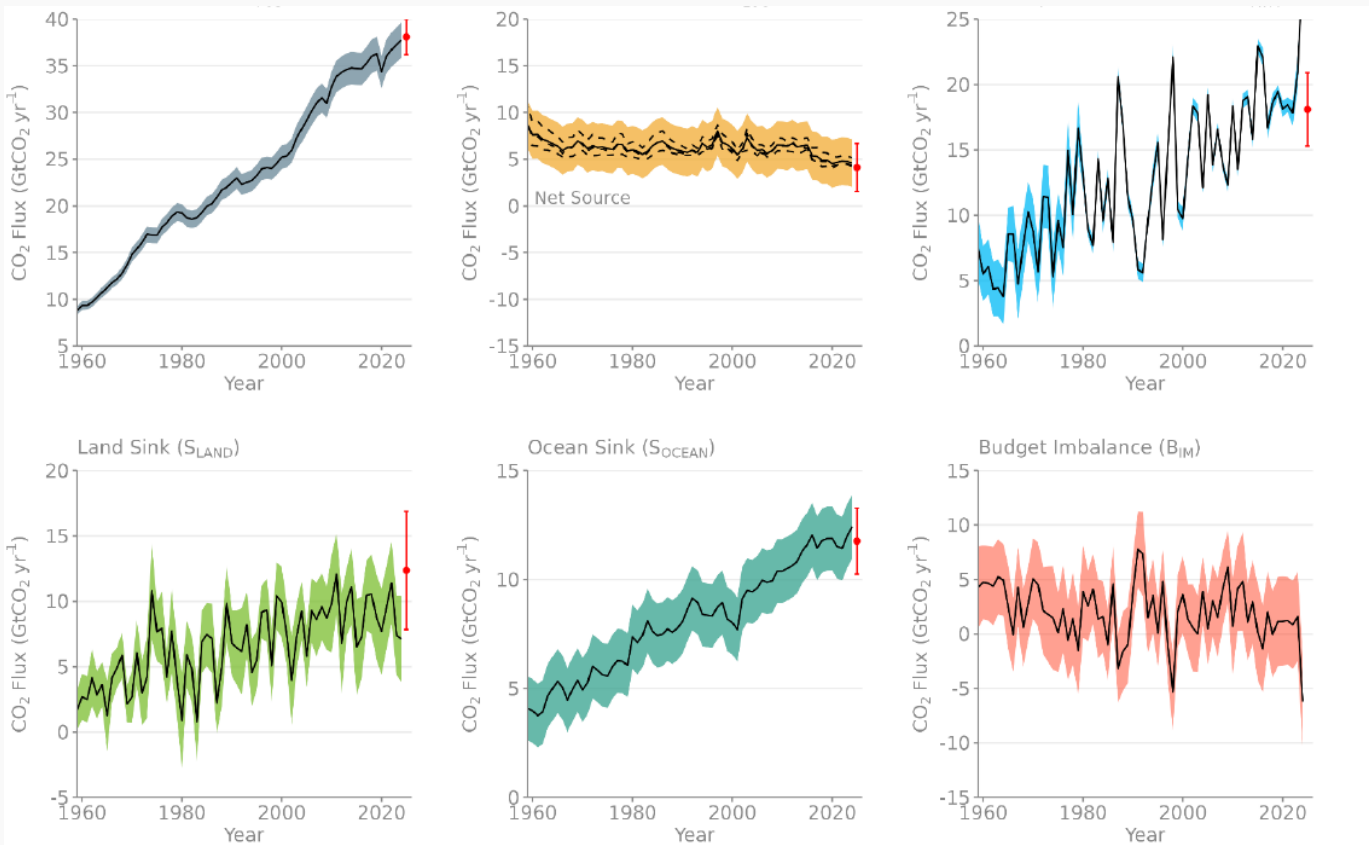

GHG emissions $\rightarrow$ atmospheric GHG concentrations

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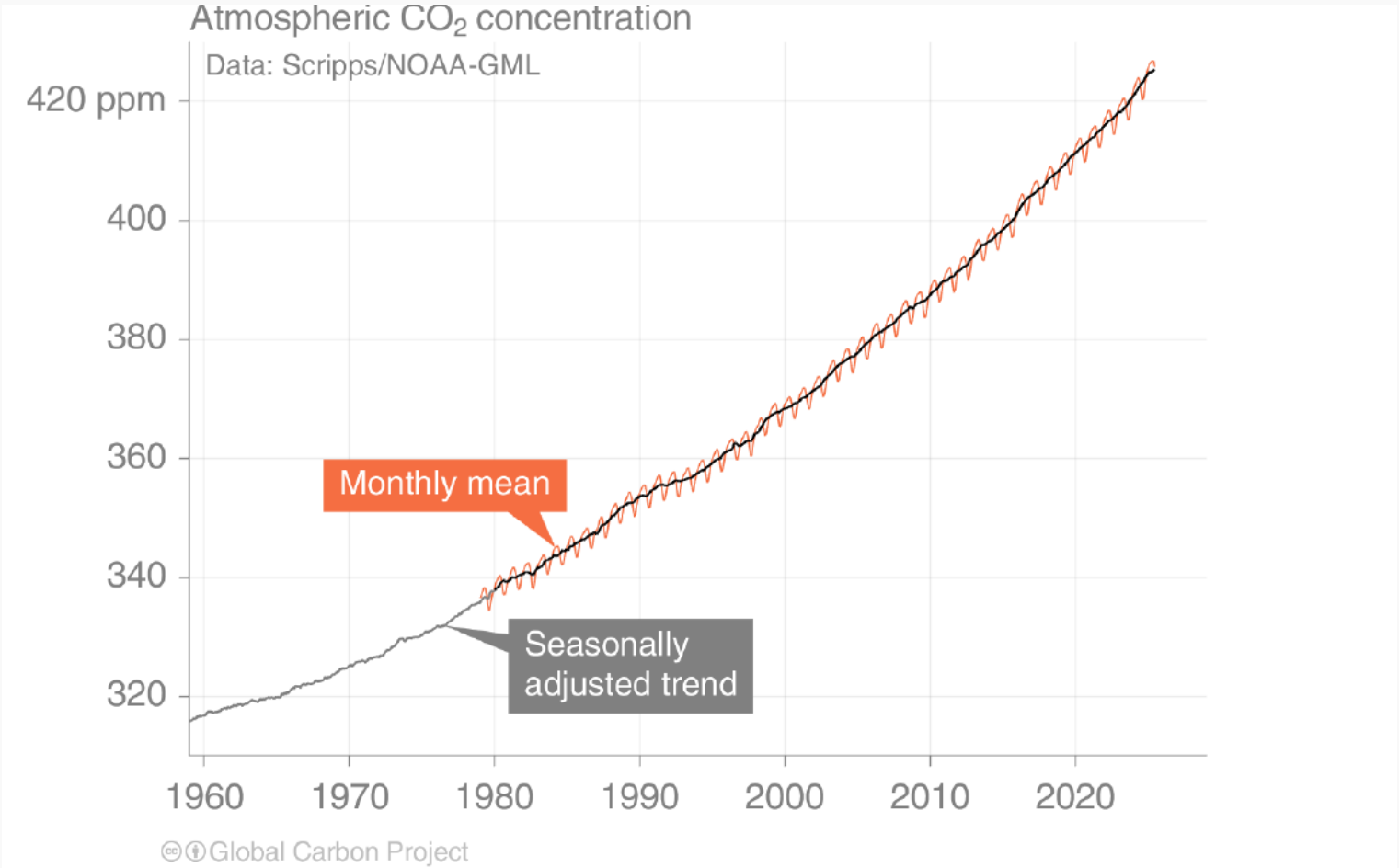

GHG emissions → atmospheric GHG concentrations

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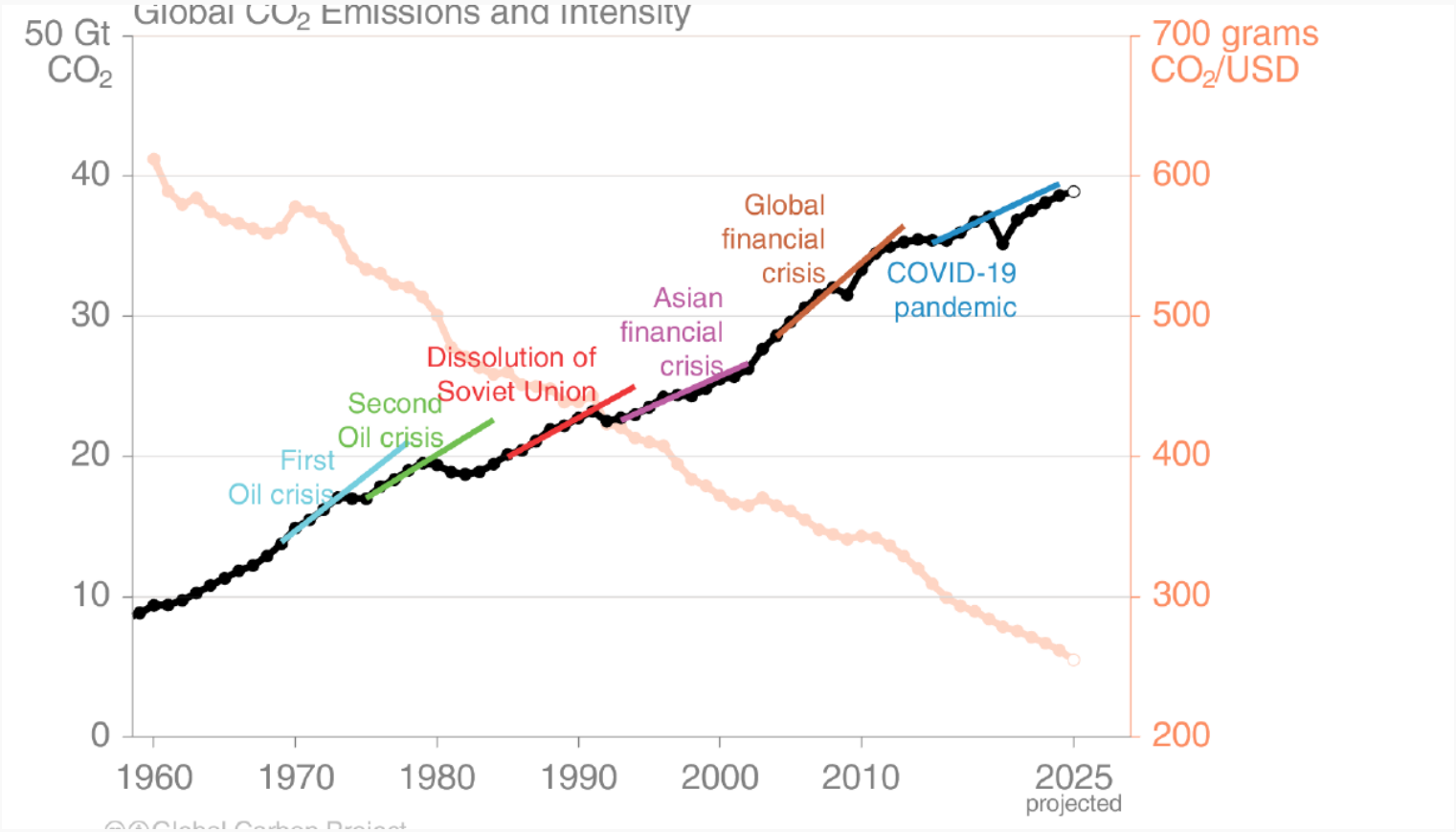


GHG emissions $\rightarrow$ atmospheric GHG concentrations

Global carbon budget as a state space model

Bennedsen, Hillebrand, Koopman (2023), A multivariate statistical model of the Global Carbon Budget 1959–2020, *Journal of the Royal Statistical Society: Series A* 186(1), 20–42.

State equations:

$$
\begin{aligned}
C^*_{t+1} &= C^*_t + E^*_{t+1} - S_LND^*_{t+1} - S_OCN^*_{t+1}, \\
S_LND^*_{t+1} &= c_1 + \frac{\beta_1}{C_{1750}} C^*_{t+1} + \beta_3 SOI_{t+1}, \\
S_OCN^*_{t+1} &= c_2 + \frac{\beta_2}{C_{1750}} C^*_{t+1} + \beta_4 SOI_{t+1}, \\
E^*_{t+1} &= E^*_t + \beta_5 \Delta \log GDP_{t+1}.
\end{aligned}
$$

GHG emissions $\rightarrow$ atmospheric GHG concentrations

Global carbon budget as a state space model

Bennedsen, Hillebrand, Koopman (2023), A multivariate statistical model of the Global Carbon Budget 1959–2020, *Journal of the Royal Statistical Society: Series A* 186(1), 20–42.

Measurement equations:

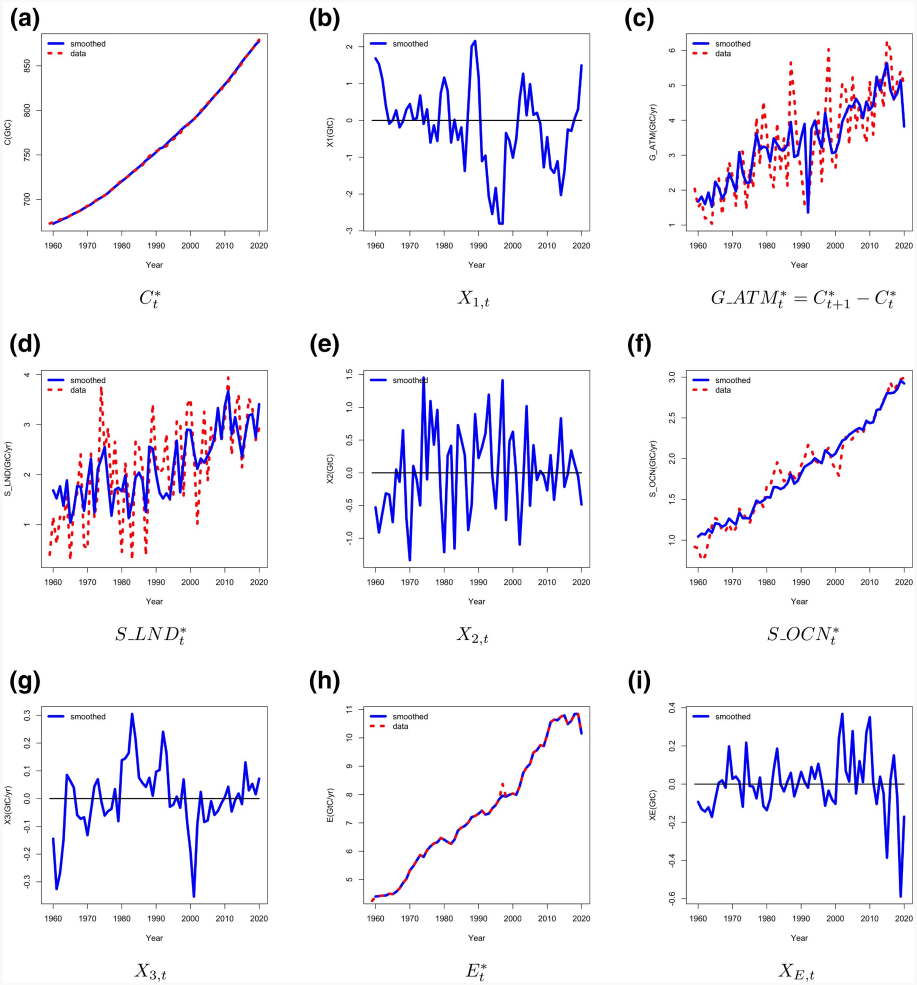
$$
\begin{aligned}
C_t &= C_t^* + X_{1,t}, \\
S_LND_t &= S_LND_t^* + X_{2,t}, \\
S_OCN_t &= S_OCN_t^* + X_{3,t}, \\
E_t &= E_t^* + X_{4,t}.
\end{aligned}
$$

Add to state equations:

$$
X_{i,t+1} = \phi_i X_{i,t} + \varepsilon_{i,t}, \quad i = 1, 2, 3, 4.
$$

GHG emissions $\rightarrow$ atmospheric GHG concentrations

Global carbon budget as a state space model



C_t^* $X_{1,t}$ $G_ATM_t^* = C_{t+1}^* - C_t^*$

$S_LND_t^*$ $X_{2,t}$ $S_OCN_t^*$

$X_{3,t}$ E_t^* $X_{E,t}$

GHG emissions $\rightarrow$ atmospheric GHG concentrations

The Global Carbon Budget as a cointegrated system

Bennedsen, Hillebrand, Nielsen (2026), The Global Carbon Budget as a cointegrated system, *The Econometrics Journal*, forthcoming.

Four annual GCB series, $Y_t = (S_t^L,\ S_t^O,\ E_t,\ C_t)'$: land sink, ocean sink, emissions, atmospheric concentration.

Structural equations:

$$\begin{aligned}
\text{sinks:} \quad & S_t^L = a_1 + b_1 C_t + X_{1,t}, \qquad S_t^O = a_2 + b_2 C_t + X_{2,t}, \\
\text{emissions:} \quad & E_t = d + E_{t-1} + X_{3,t}, \\
\text{budget:} \quad & C_t = C_{t-1} + E_t - S_t^L - S_t^O + X_{4,t},
\end{aligned}$$

with AR(1) deviations $X_{i,t} = \phi_i X_{i,t-1} + \varepsilon_{i,t}$, $i = 1, \ldots, 4$, and climate covariates $Z_t = (\mathrm{ENSO}_t,\ \mathrm{PDO}_t,\ \mathrm{COVID}_t)$.

GHG emissions $\rightarrow$ atmospheric GHG concentrations

A structural vector error-correction model

Bennedsen, Hillebrand, Nielsen (2026), *The Econometrics Journal*, forthcoming.

Stacking (2.1)–(2.3) gives the structural form

$$A_0 \, \Delta Y_t = c + C \, Z_t + \Pi \, Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + \varepsilon_t, \qquad A_0 = \begin{bmatrix} 1 & 0 & 0 & -b_1 \\ 0 & 1 & 0 & -b_2 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 1 \end{bmatrix}.$$

- A_0 collects the contemporaneous physical relations: the sink equations (rows 1–2, sinks respond to C_t) and the carbon-budget identity (row 4).
- Structural shocks ε_t are contemporaneously uncorrelated (diagonal covariance).
- Three cointegrating relations $\beta' Y_t$ (two sink relations and the budget); rank $r = 3$, so a single common stochastic trend, given by emissions.
- The reduced-form VECM follows by left-multiplying with A_0^{-1} (equivalent, not shown).

GHG emissions $\rightarrow$ atmospheric GHG concentrations

Estimation and testing

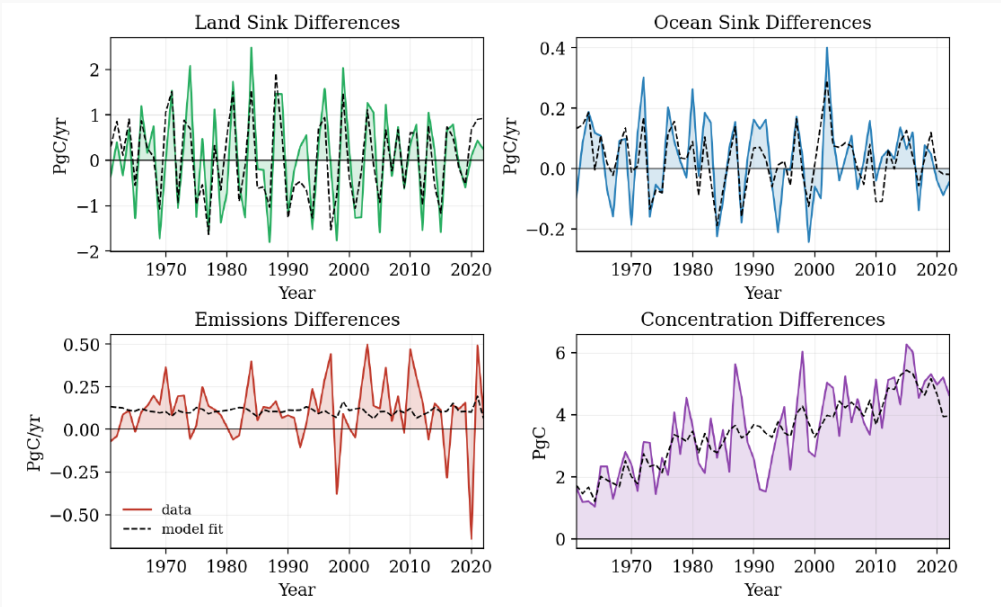
Bennedsen, Hillebrand, Nielsen (2026), *The Econometrics Journal*, forthcoming.

- Lag length: $k = 1$ (information criteria and an LR test of lag exclusion).
- Cointegration rank: the Johansen trace test gives $r = 3$ (bootstrap p-values); one common stochastic trend.
- Exclusion and weak exogeneity (LR tests): emissions E_t cannot be excluded, yet its adjustment row in α is zero. Emissions is weakly exogenous, the single driver; the sinks and concentration adjust to disequilibrium.
- Restricted (structural) vs. unrestricted model: $\mathrm{LR} = 33.19$, χ^2_{36}, $p = 0.60$. The physical restrictions are not rejected, and we adopt the restricted model.

GHG emissions $\rightarrow$ atmospheric GHG concentrations

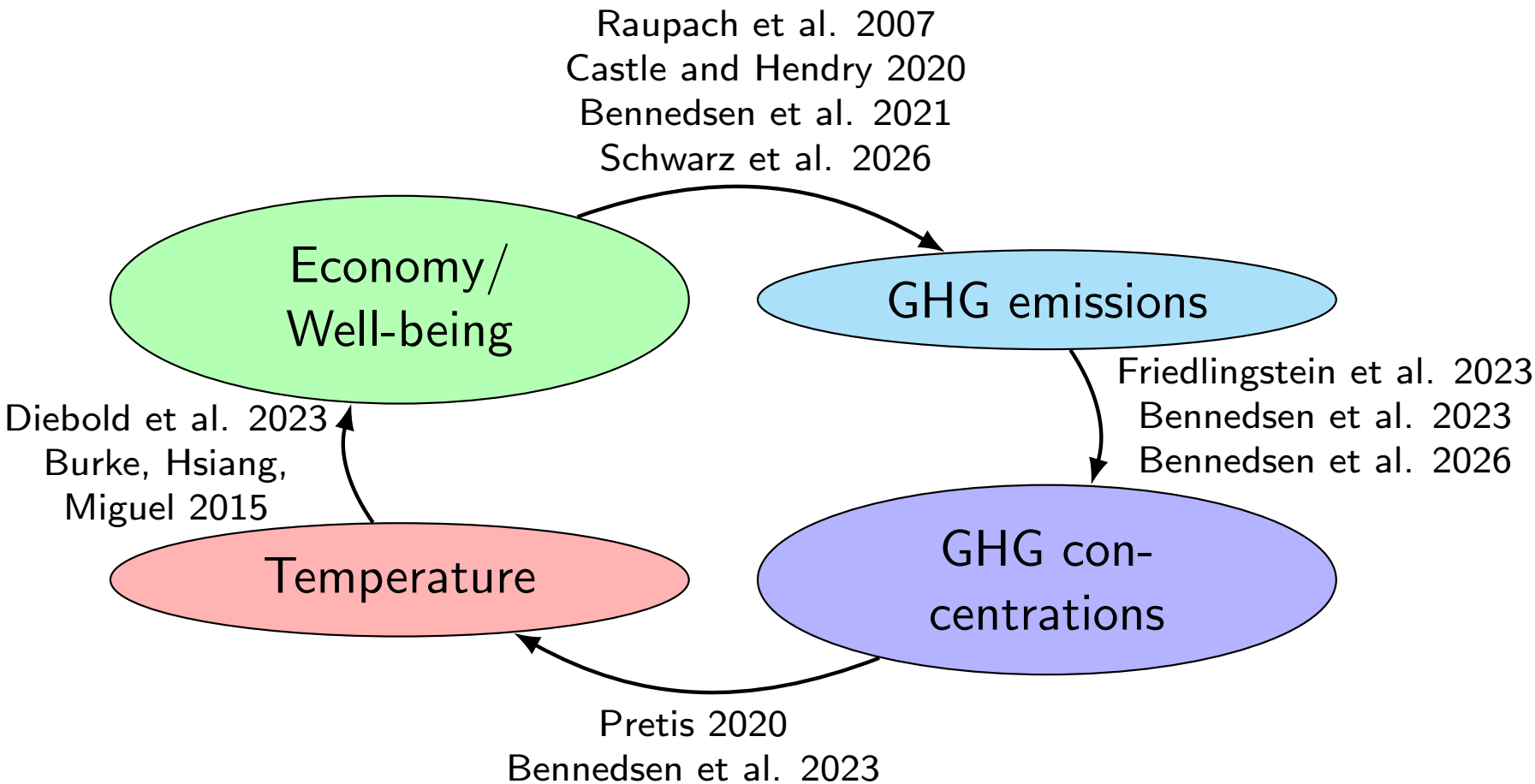
Fit, diagnostics, and projections to 2100

Bennedsen, Hillebrand, Nielsen (2026), *The Econometrics Journal*, forthcoming. Figure 3: first differences, data vs. model fit.



- Residual diagnostics show no misspecification (COVID-2020 dummy needed for Gaussianity).
- Used for projections of C_t, S_t^L, S_t^O to 2100 under SSP scenarios, consistent with reduced-complexity climate models.

Last edge


Raupach et al. 2007
Castle and Hendry 2020
Bennedsen et al. 2021
Schwarz et al. 2026
Economy/
Well-being
GHG emissions
Friedlingstein et al. 2023
Bennedsen et al. 2023
Bennedsen et al. 2026
Diebold et al. 2023
Burke, Hsiang,
Miguel 2015
Temperature
GHG con-
centrations
Pretis 2020
Bennedsen et al. 2023

Economy $\rightarrow$ GHG emissions

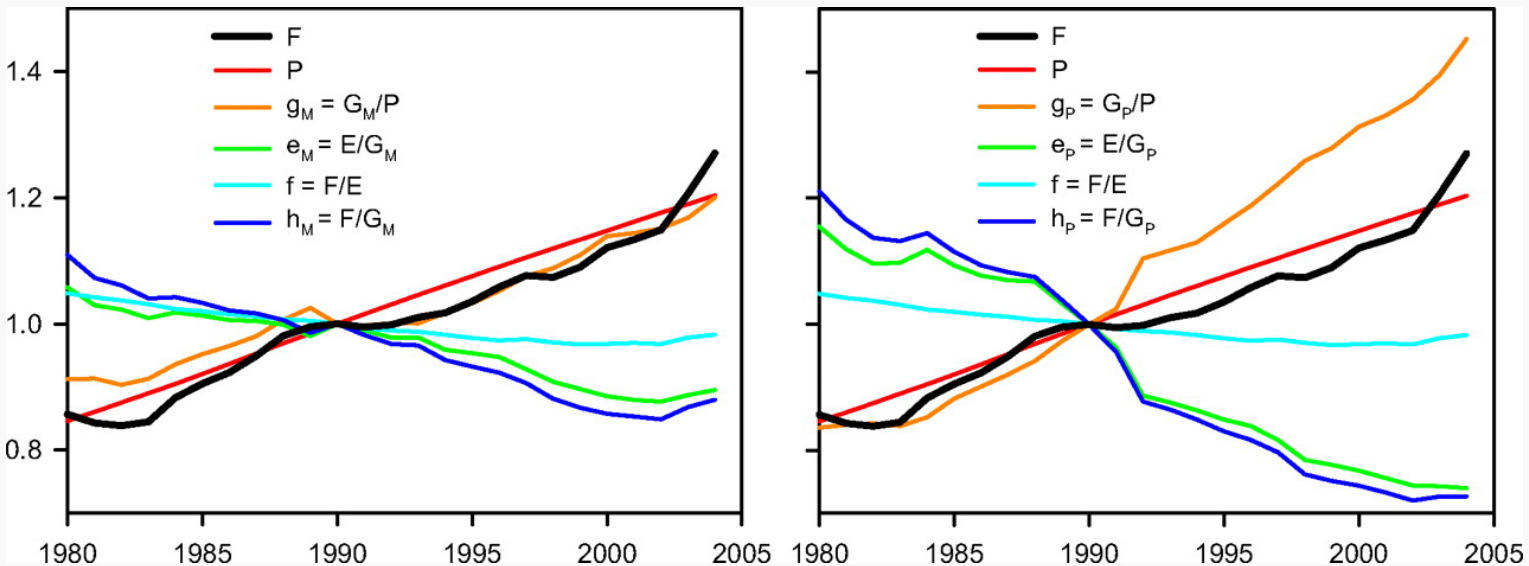
Kaya identity

Raupach et 6 al. (2007), Global and regional drivers of accelerating CO_2 emissions, *PNAS* 104(24), 10288–10293

$$\begin{aligned}
\text{Emissions} &= \text{Population} \; \frac{\text{GDP}}{\text{Population}} \; \frac{\text{Energy}}{\text{GDP}} \; \frac{\text{Emissions}}{\text{Energy}} \\
&= \text{Population} \times \text{per-capita GDP} \times \text{energy-intensity of GDP} \times \text{emissions intensity of energy} \\
&= P \; g \; e \; f
\end{aligned}$$

Economy $\rightarrow$ GHG emissions

Kaya identity



Economy $\rightarrow$ GHG emissions

General-to-specific modeling with index-indicator saturation

Castle and Hendry (2020), Climate Econometrics: An Overview, Foundations, and Trends, *Foundations and Trends in Econometrics* 10 (3-4), 145–322.

Let E be annual CO_2 emissions in the U.K. 1861–2011, C be coal use, O oil use, k log of capital stock, g log of real GDP, S step indicators selected by *Autometrics*. Then, the fitted long-run relation in a cointegration estimation is

$$\begin{aligned}\widetilde{E}_{LR} = & \underset{(0.06)}{2.0}\, C + \underset{(0.18)}{1.4}\, O + \underset{(0.27)}{1.18}\, k - \underset{(0.28)}{0.27}\, g + \underset{(6)}{63}\, S_{\{1924\}} \\ & - \underset{(14)}{64.0}\, S_{\{1968\}} + \underset{(13)}{70}\, S_{\{2009\}} - \underset{(165)}{328.} \end{aligned} \tag{7.5}$$

Economy → GHG emissions

General-to-specific modeling with index-indicator saturation

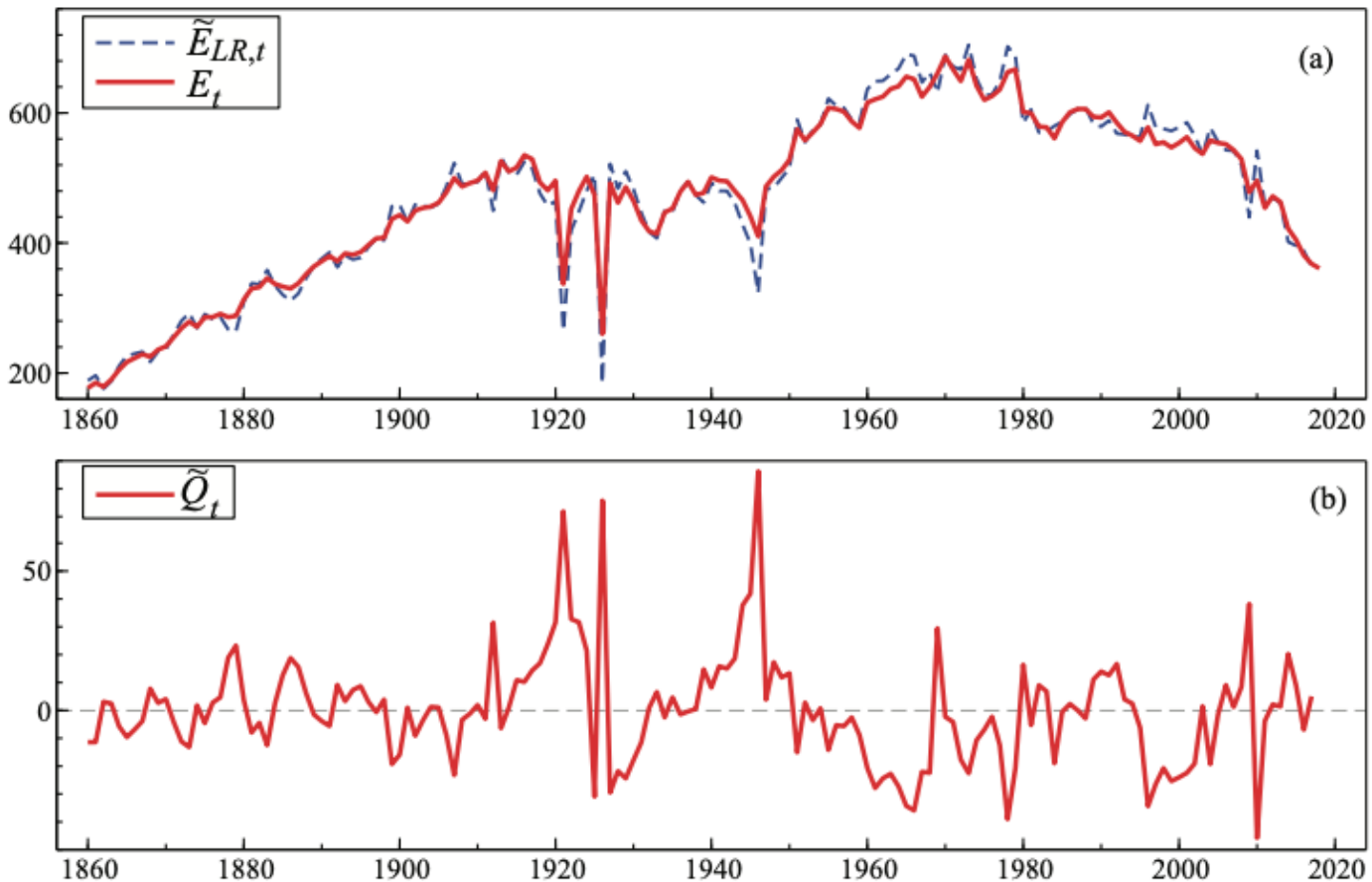


Figure 7.7: (a) E_t and $\widetilde{E}_{LR,t}$; (b) $\widetilde{Q}_t = E_t - \widetilde{E}_{LR,t}$ centered on a mean of zero.

Economy $\rightarrow$ GHG emissions

General-to-specific modeling with index-indicator saturation

The error-correction representation is estimated as

$$\begin{aligned}\widehat{\Delta E}_t = & \underset{(0.10)}{1.88}\ \Delta C_t + \underset{(0.21)}{1.71}\ \Delta O_t + \underset{(1.09)}{7.15}\ \Delta k_t + \underset{(0.28)}{0.89}\ \Delta g_t \\ & - \underset{(0.05)}{0.50}\ \widetilde{Q}_{t-1} - \underset{(2.4)}{15.2} - \underset{(8.8)}{79.4}\ \Delta 1_{\{1926\}} + \underset{(6.4)}{50.2}\ \Delta 1_{\{1947\}} \\ & - \underset{(11.1)}{45.8}\ 1_{\{1921\}} - \underset{(8.9)}{27.5}\ 1_{\{1912\}} + \underset{(8.9)}{26.8}\ 1_{\{1978\}} + \underset{(8.9)}{28.4}\ 1_{\{1996\}} \end{aligned} \tag{7.6}$$

$$\begin{aligned} \widehat{\sigma} = & 8.87 \ \ \mathsf{R}^2 = 0.94 \ \ \mathsf{F}_{\mathsf{AR}}(2,139) = 0.49 \ \ \chi^2_{\mathsf{nd}}(2) = 1.67 \\ & \mathsf{F}_{\mathsf{Het}}(14,134) = 1.03 \ \ \mathsf{F}_{\mathsf{ARCH}}(1,151) = 0.53 \\ & \mathsf{F}_{\mathsf{Reset}}(2,139) = 1.50 \ \ \mathsf{F}_{\mathsf{nl}}(15,126) = 1.35 \ \ \mathsf{F}_{\mathsf{Chow}}(4,141) = 1.75. \end{aligned}$$

Economy $\rightarrow$ GHG emissions

Macroeconomic factor models

Bennedsen, Hillebrand, Koopman (2021), Modeling, forecasting, and nowcasting U.S. CO_2 emissions using many macroeconomic predictors, *Energy Economics* 96, 105118

Let E_t be annual CO_2 emissions in the U.S., let f_t be a set of macroeconomic factors, let z_t be exogenous variables such as dummies. We model

$$\Delta \log E_t = \alpha + b_1 IP_t + b_2 IP_t^{\text{res.utils.}} + \beta^T f_t + \gamma^T z_t + \varepsilon_{y,t}.$$

The two IP variables are pre-selected from the set x_t of macroeconomic variables (transformed to stationarity), from which the factors are determined:

$$\begin{aligned} x_t &= \Gamma f_t + \varepsilon_{x,t}, \\ f_t &= \Phi f_{t-1} + \eta_t. \end{aligned}$$

The selection can be done using different methods, we used goodness of fit in regressions of $\Delta \log E_t$ on two regressors from x_t. LASSO, elastic net, and Autometrics are alternatives.

If one defines $\tilde{f} = (IP, IP^{\text{res.utils.}}, f)$, one can consider our procedure as a factor extraction that is *supervised* for the forecast target $\Delta \log E$.

Economy $\rightarrow$ GHG emissions

Macroeconomic factor models

Bennedsen, Hillebrand, Koopman (2021), Modeling, forecasting, and nowcasting U.S. CO_2 emissions using many macroeconomic predictors, *Energy Economics* 96, 105118

Model can be written as

$$\begin{bmatrix} \Delta \log E_t \\ x_t \end{bmatrix} = \begin{bmatrix} \alpha \\ 0 \end{bmatrix} + \begin{bmatrix} \beta^T + b^T \Gamma \\ \Gamma \end{bmatrix} f_t + \begin{bmatrix} \gamma^T \\ 0 \end{bmatrix} z_t + u_t,$$

$$f_t = \Phi f_{t-1} + \eta_t.$$

$$b = (0, \ldots, 0, b_1, 0, \ldots, 0, b_2, 0, \ldots)^T$$

Parameter vector $(\alpha, b, \beta, \gamma, \Gamma, \Phi, \Sigma)$, where Σ contains all variances. Two-stage procedure of Doz, Giannone, Reichlin (2012), A quasi-maximum likelihood approach for large, approximate dynamic factor models, *Review of Economics and Statistics* 94(4), 1014–1024.

1. Run principal component analysis of x_t. Use principal components as $\hat{f}^{(1)}$, store $\hat{\Gamma}^{(1)}$.
2. Run regression of $\hat{f}_t^{(1)}$ on $\hat{f}_{t-1}^{(1)}$. Gives $\hat{\Phi}^{(1)}$.
3. Estimate remaining elements of parameter vector $(\alpha, b, \beta, \gamma, \Sigma)$ by maximum likelihood and Kalman filter.
4. Plug everything into model equation and run MLE until convergence. Use smoothed values for $\hat{f}_t$.

Economy $\rightarrow$ GHG emissions

Macroeconomic factor models

Bennedsen, Hillebrand, Koopman (2021), Modeling, forecasting, and nowcasting U.S. CO_2 emissions using many macroeconomic predictors, *Energy Economics* 96, 105118

Two flavors: $\beta \neq 0$ (U) and $\beta = 0$ (R). Initial regression of $\Delta \log E_t$ on IP and $IP^{\text{res.utils.}}$ yields $R^2 \approx 70\%$.

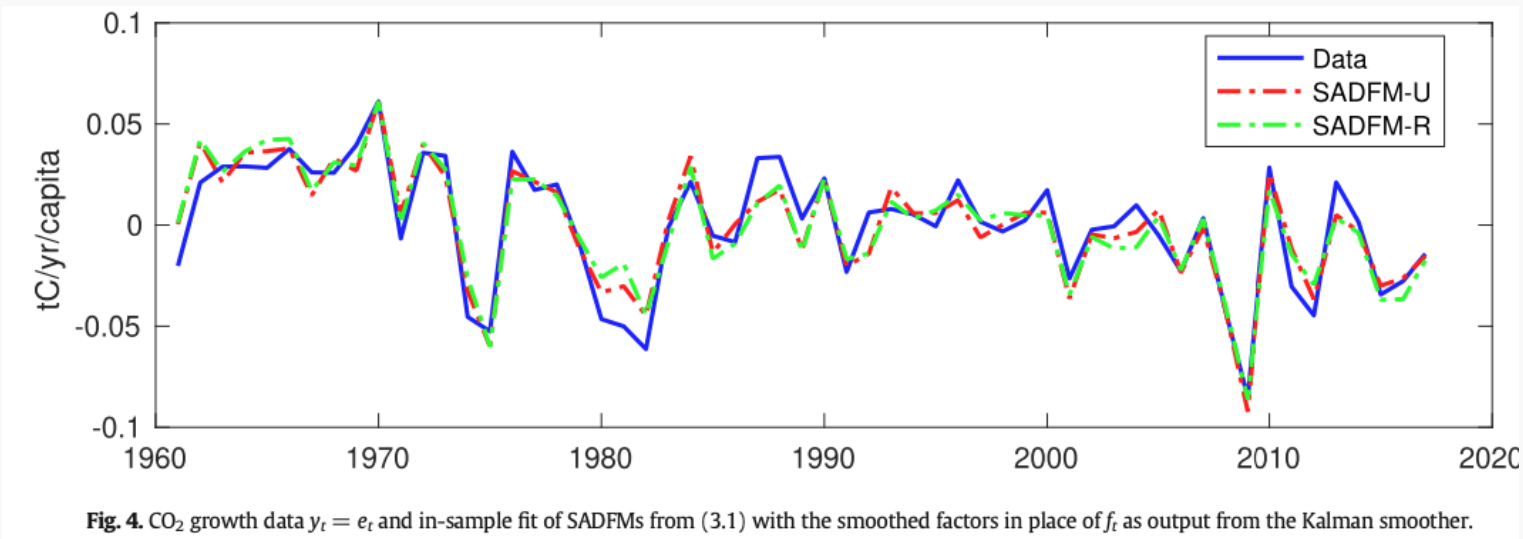


Fig. 4. CO_2 growth data $y_t = e_t$ and in-sample fit of SADFMs from (3.1) with the smoothed factors in place of f_t as output from the Kalman smoother.

Economy → GHG emissions

Macroeconomic factor models

Bennedsen, Hillebrand, Koopman (2021), Modeling, forecasting, and nowcasting U.S. CO_2 emissions using many macroeconomic predictors, *Energy Economics* 96, 105118

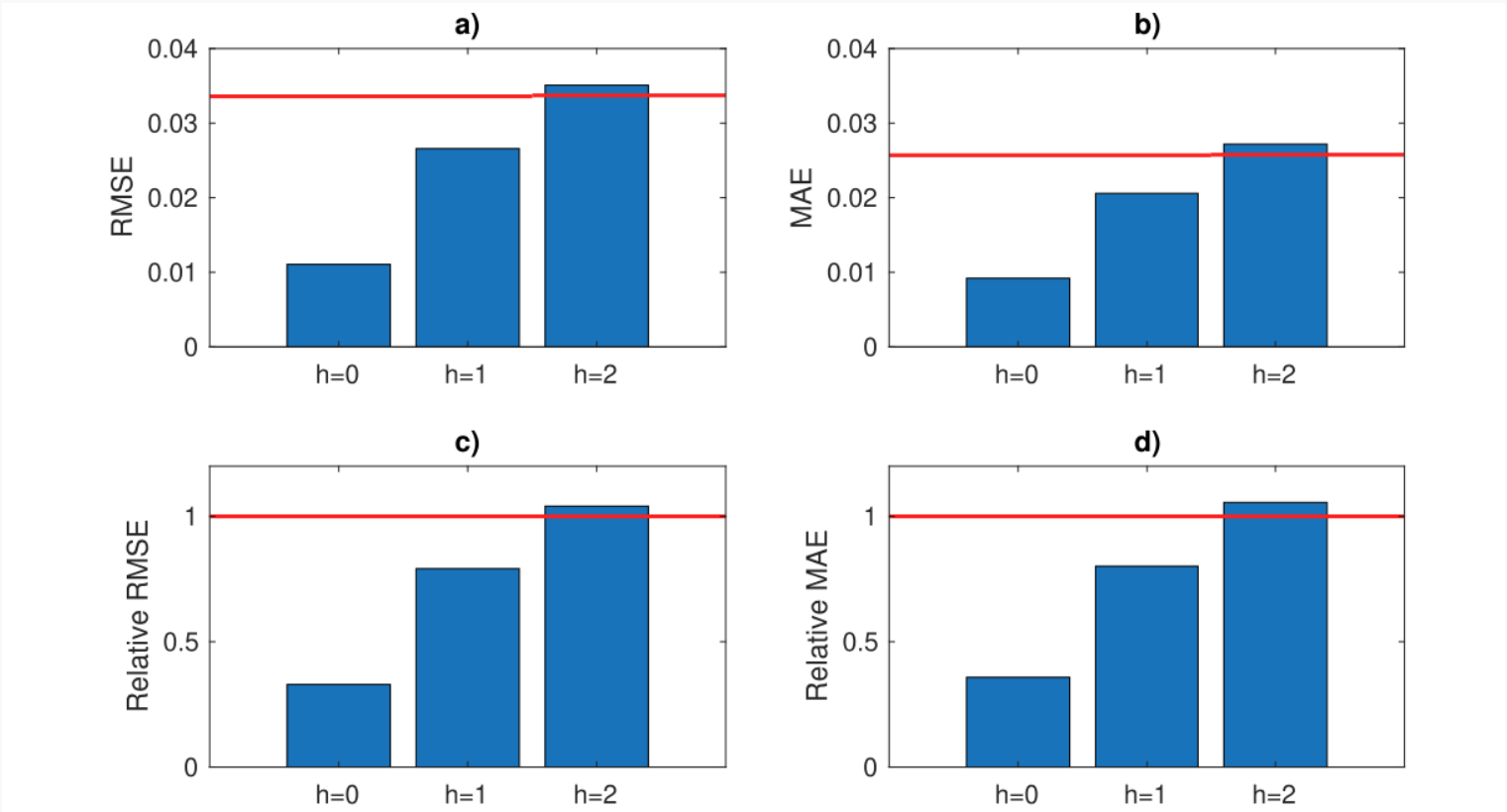


Fig. 6. Out-of-sample forecasting h years ahead for $h = 0,1,2$. $h = 0$ is "nowcasting". a): Root mean squared error (RMSE) of SADFM-R. b): Mean absolute error (MAE) of SADFM-R. c): RMSE of SADFM-R as fraction of the RMSE of the constant growth model. d): MAE of SADFM-R as fraction of the RMSE of the constant growth model. The — red line shows the performance of the constant growth model. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Economy → GHG emissions

Macroeconomic factor models

Bennedsen, Hillebrand, Koopman (2021), Modeling, forecasting, and nowcasting U.S. CO_2 emissions using many macroeconomic predictors, *Energy Economics* 96, 105118

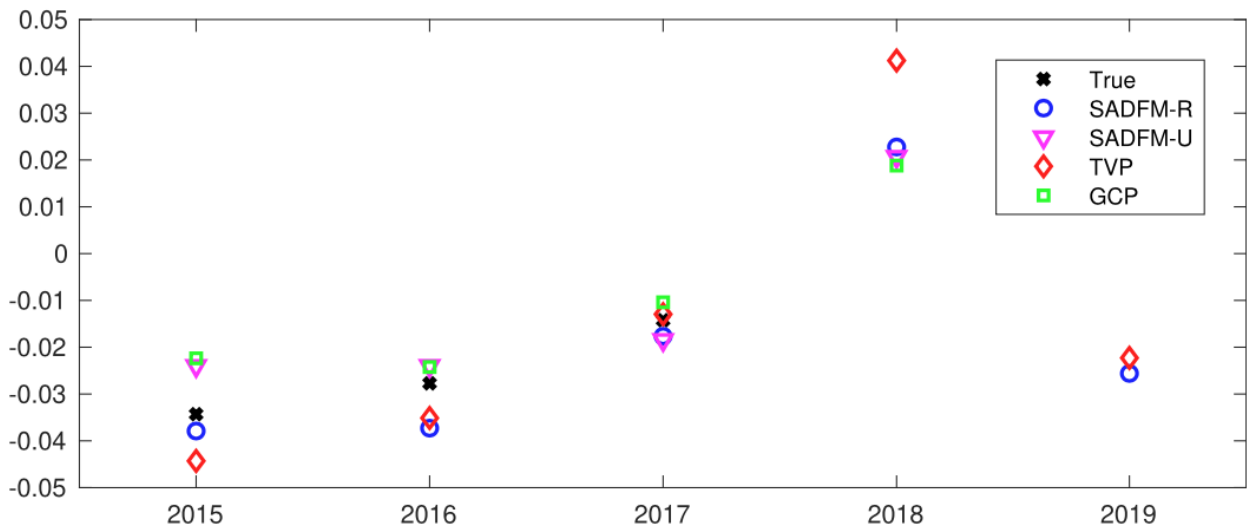


Fig. 7. Nowcasting U.S. CO_2 emissions per capita growth. Black crosses: True growth rates. Blue circles: Nowcasts from SADFM-R. Magenta triangles: Nowcasts from SADFM-U. Red diamond: Nowcasts from TVP model (see Appendix 10). Green square: Nowcasts from the Global Carbon Project. At the time of writing, official U.S. CO_2 emissions for 2018 and 2019 were not available. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Economy → GHG emissions

OSEM: an empirical macro model for emissions

Schwarz, Kurle, Pretis, Martinez, Harper (2026), An Open-Source Empirical Macroeconomic (OSEM) Model for Economic and Emission Forecasting, *International Journal of Forecasting*, forthcoming.

- Open-source empirical macro model; short-run (1–5 year) sectoral CO_2 forecasts.
- Data-driven empirical Integrated Assessment Model, set against closed-source policy models and long-run cost-effectiveness IAMs.
- Modular system of national-accounts identities and estimated regression modules.
- Distributed as the `R` package `osem`; ported to another country in one line.

Inspired by the Norwegian Aggregate Model (Nymoen and Bårdsen 2023).

Economy $\rightarrow$ GHG emissions

Accounting identities as the backbone

Schwarz, Kurle, Pretis, Martinez, Harper (2026), OSEM, *IJF*, forthcoming.

Supply equals demand pins down the system:

$$\text{Supply} = \text{GDPOutput} + \text{Imports}, \qquad \text{Demand} = \text{GDPExpenditure} + \text{Imports}.$$

Output and expenditure decompositions:

$$\begin{aligned} \text{GDPOutput} &= \textstyle\sum_j \text{VASector}_j + \text{Taxes} - \text{Subsidies}, \\ \text{GDPExpenditure} &= \text{ConsHH} + \text{ConsGov} + \text{CapForm} \\ &\quad + \text{Exports} - \text{Imports} + \text{DInventories}. \end{aligned}$$

Inventories close the system as an identity:

$$\text{DInventories} = \text{Supply} - \text{ConsHH} - \text{ConsGov} - \text{CapForm} - \text{Exports}.$$

Economy → GHG emissions

From an emissions identity to estimated modules

Schwarz, Kurle, Pretis, Martinez, Harper (2026), OSEM, *IJF*, forthcoming.

Decompose the target into a modelled and an unmodelled part:

$$\mathrm{CO}_{2,t}^{\mathrm{total}} = \mathrm{CO}_{2,t}^{\mathrm{ind}} + \mathrm{CO}_{2,t}^{\mathrm{other}}.$$

Model industry emissions on real activity and prices (one estimated module):

$$\begin{aligned}\mathrm{CO}_{2,t}^{\mathrm{ind}} = b_{32}\,\mathrm{VA}_{t}^{\mathrm{ind}} + a_{32}\,\mathrm{VA}_{t-1}^{\mathrm{ind}} + a_{33}\,\mathrm{CO}_{2,t-1}^{\mathrm{ind}} \\ + c_{31}^{0}\mathrm{HICP}_{t} + c_{31}^{1}\mathrm{HICP}_{t-1} + e_{3,t}.\end{aligned}$$

Value added and household consumption enter as their own modules; every module carries contemporaneous and lagged terms.

Economy → GHG emissions

A Cholesky-identified SVAR

Schwarz, Kurle, Pretis, Martinez, Harper (2026), OSEM, *IJF*, forthcoming.

Stack the modules with a lower-triangular B, ordering $Y_t = (\mathrm{Cons}_t^{\mathrm{hh}},\ \mathrm{VA}_t^{\mathrm{ind}},\ \mathrm{CO}_{2,t}^{\mathrm{ind}})'$:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -b_{21} & 1 & 0 \\ 0 & -b_{32} & 1 \end{bmatrix}}_{B} Y_t = A\,Y_{t-1} + C_0 X_t + C_1 X_{t-1} + e_t.$$

Invert B to get the reduced-form VAR with exogenous regressors X:

$$Y_t = B^{-1} A\,Y_{t-1} + B^{-1} C_0 X_t + B^{-1} C_1 X_{t-1} + B^{-1} e_t.$$

Recursive ordering $\mathrm{Cons}^{\mathrm{hh}} \to \mathrm{VA}^{\mathrm{ind}} \to \mathrm{CO}_2^{\mathrm{ind}}$: contemporaneous effects run one way, feedback only with a lag. Equation-by-equation OLS equals an SVAR identified by sequential (Cholesky) restrictions.

Economy → GHG emissions

The full specification: 42 modules

Schwarz, Kurle, Pretis, Martinez, Harper (2026), OSEM, *IJF*, forthcoming.

Identities	Supply, GDPOutput, GDPExpenditure, DInventories, real = nominal / deflator
Estimated (emissions)	EmiCO2ManInd = f(RealVAIndustry, HICPlocal) EmiCO2ElecHeat = f(ElectrCons, RealVAIndustry, PriceETS, CDD, HDD) EmiCO2RoaTra = f(RoadFreight, RealConsHH, PriceOil, HICPDiesel) ElectrCons = f(RealConsHH, HICP_Electricity, PriceETS)

Emissions depend on *real* activity; nominal variables are converted by deflator identities.

Economy → GHG emissions

Automated data pipeline

Schwarz, Kurle, Pretis, Martinez, Harper (2026), OSEM, *IJF*, forthcoming.

- Variable dictionary maps model names to public databases: Eurostat, IMF, Statistics Canada, EDGAR (emissions).
- Oil price from FRED; EU-ETS price from ICAP / EEX.
- Frequencies matched to quarterly: flows summed, price indices (HICP) averaged.
- Re-estimated as data is released; ported to another country by changing the country filter.

Economy → GHG emissions

General-to-specific estimation

Schwarz, Kurle, Pretis, Martinez, Harper (2026), OSEM, *IJF*, forthcoming.

- Each module selected by general-to-specific search (`gets`): covariates and lag length.
- Impulse- and step-indicator saturation (IIS / SIS) detect outliers, location shifts, and trend breaks.
- Estimated in levels (ARDL) or in equilibrium-correction form.
- Jointly cointegrated modules estimated as a CVAR (Johansen trace test).

Same indicator-saturation machinery as the Castle and Hendry (2020) example above.

Economy → GHG emissions

Forecasting the system

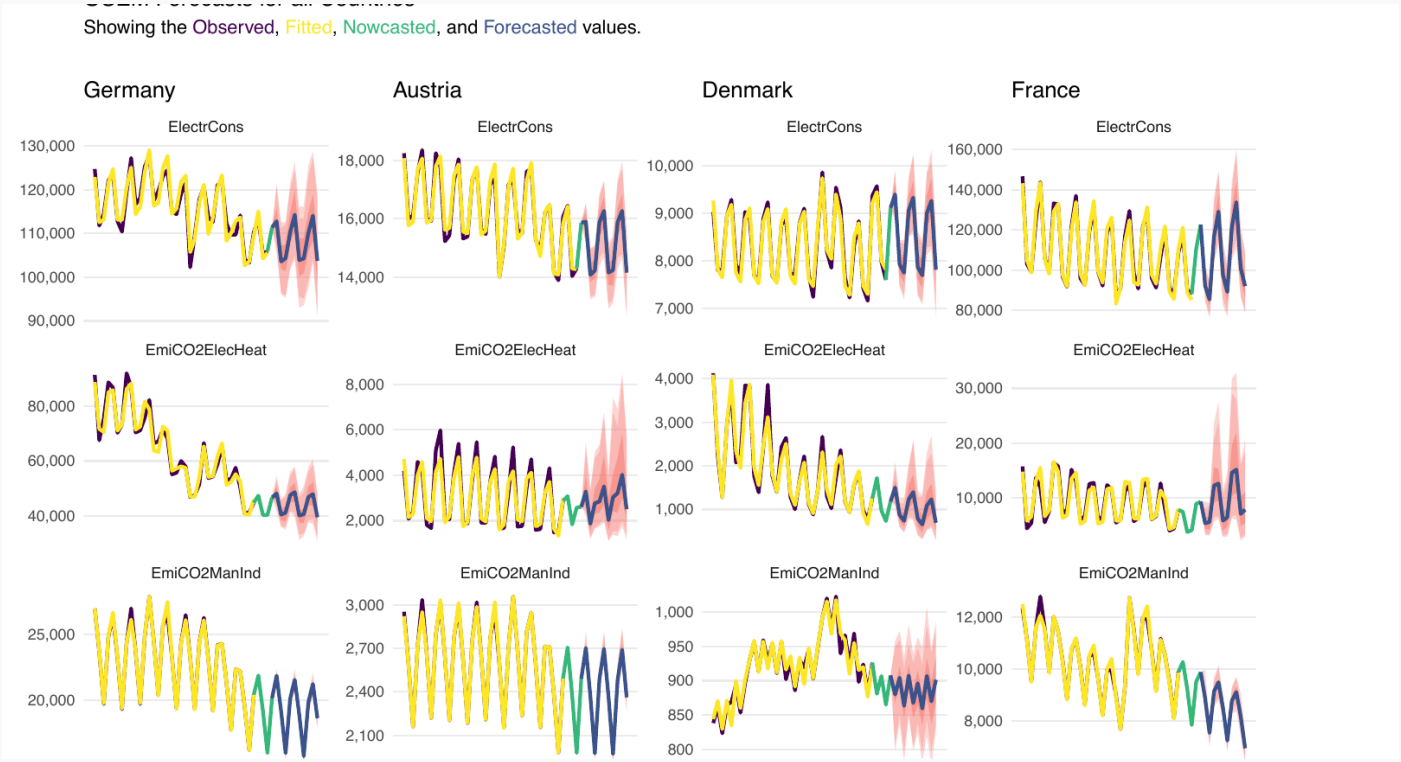
Schwarz, Kurle, Pretis, Martinez, Harper (2026), OSEM, *IJF*, forthcoming.

- Sequential ordering: forecast the lowest aggregation first, feed it upward automatically.
- Exogenous paths from built-in devices (AR with seasonals, ARIMA, exponential smoothing) or user scenarios, e.g. a prescribed carbon-price path.
- Nowcasting to a common forecast origin across ragged-edge series.
- Forecast bands by residual bootstrap, propagated module by module.

Economy → GHG emissions

Forecasts for four countries

Schwarz, Kurle, Pretis, Martinez, Harper (2026), OSEM, *IJF*, forthcoming. Observed / fitted / nowcast / forecast, with bootstrap bands.



Economy → GHG emissions

Does it forecast? Evaluation

Schwarz, Kurle, Pretis, Martinez, Harper (2026), OSEM, *IJF*, forthcoming.

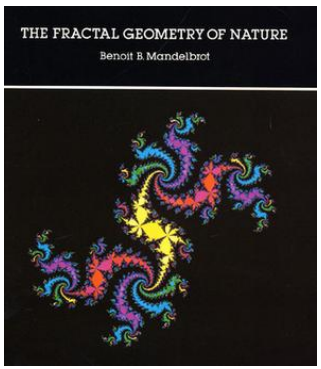
Pseudo out-of-sample (expanding window) and real-time out-of-sample against data released after submission. Mean squared prediction error, ratio of benchmark to OSEM (value > 1: benchmark worse):

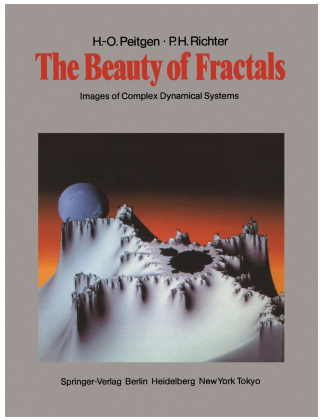
Series ($h = 1$)	RW	AR	VAR	BVAR
EmiCO2ManInd	36.2^{***}	2.93^{**}	13.5	5.77^{***}
EmiCO2ElecHeat	1.24	1.18	0.72	0.72
EmiCO2RoaTra	2.48^{*}	1.36	1.34	0.74
ElectrCons	0.24	0.15	0.32	0.09

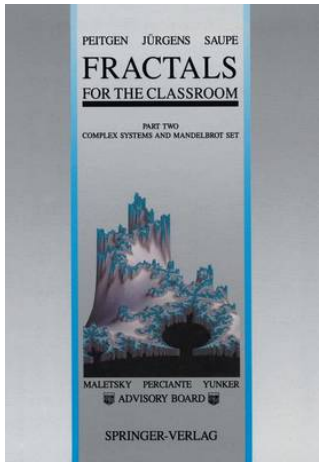
Diebold–Mariano vs OSEM: $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$. OSEM wins on the CO_2 series; electricity consumption is the exception.

2. The amazing linearity

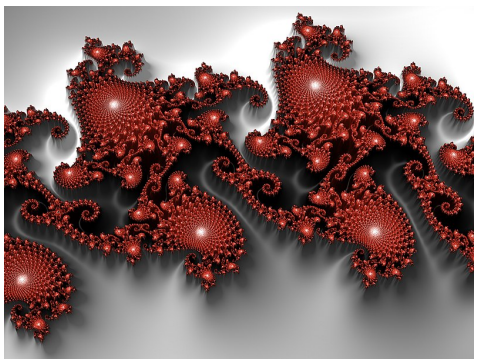
1980s: “Don’t neglect the nonlinearities!”



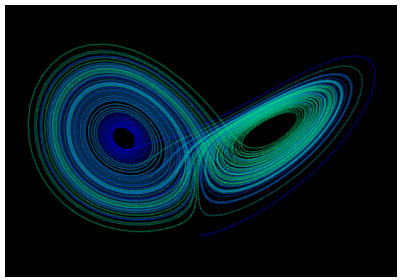






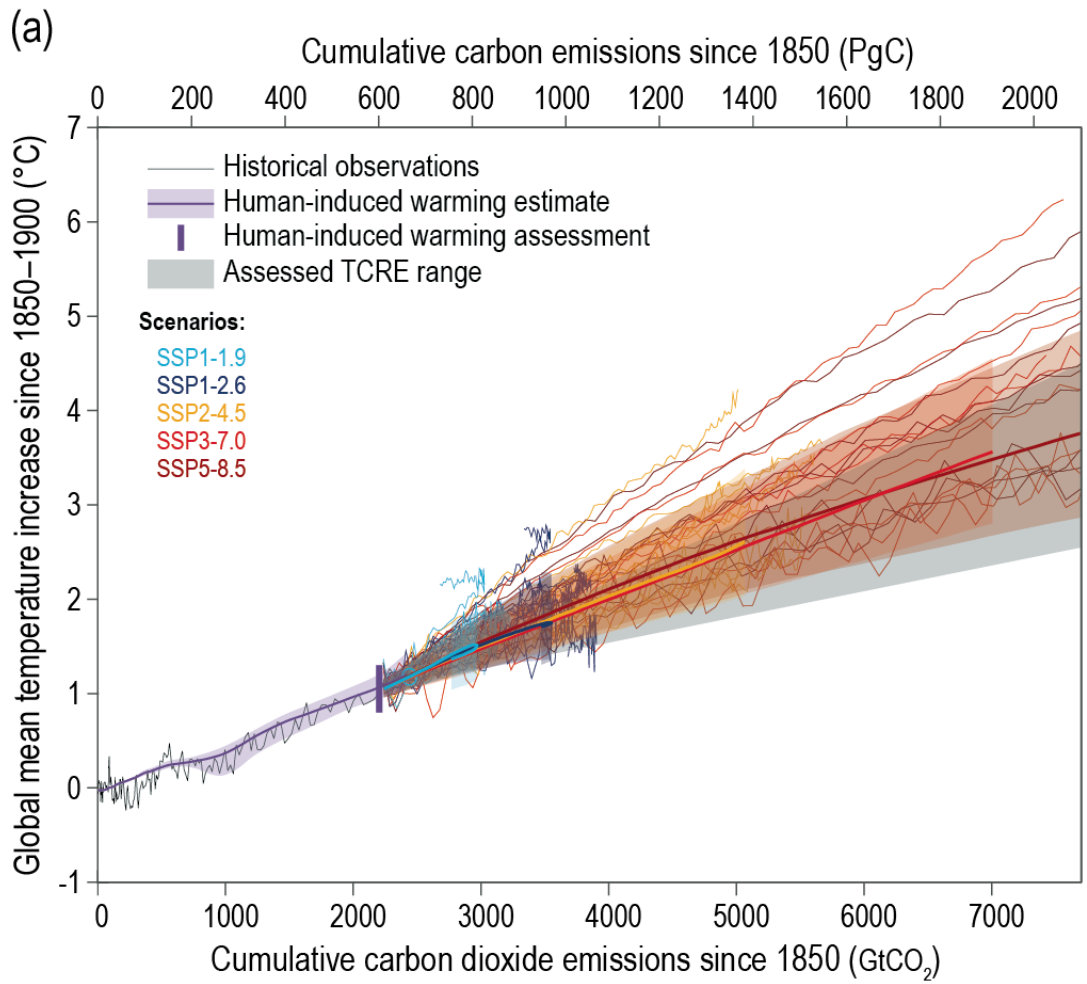








IPCC AR6 WGI Ch. 1 Technical Summary



Is this a spurious regression?

- D.P. Cummins, D.B. Stephenson, P.A. Stott, 2022, Could detection and attribution of climate change be spurious regression?, *Climate Dynamics* 59, 2785–2799, doi.org/10.1007/s00382-022-06242-z
- F. Pretis, 2020, Econometric modelling of climate systems: The equivalence of energy balance models and cointegrated vector autoregressions, *Journal of Econometrics* 214(1), 256–273, doi.org/10.1016/j.jeconom.2019.05.013
- M. Bennedsen, E. Hillebrand, S.J. Koopman, 2023, A multivariate statistical model of the global carbon budget 1959–2020, *Journal of the Royal Statistical Society Series A* 186(1), 20–42, doi.org/10.1093/jrsssa/qnac014
- D.P. Cummins, D.B. Stephenson, P.A. Stott, 2020, Optimal estimation of stochastic energy balance model parameters, *Journal of Climate* 33, 7909-7926, doi.org/10.1175/JCLI-D-19-0589.1
- M. Bennedsen, E. Hillebrand, J. Zhou Lykke, 2023, Global temperature projections from a statistical energy balance model using multiple sources of historical data, *Journal of Climate* 36(19), 6817–6838, doi.org/10.1175/JCLI-D-22-0460.1

Is this relation expected from theory?

Conceptualization of the response of a climate variable to forcing

(M. Raupach, 2013, The exponential eigenmodes of the carbon-climate system, and their implications for ratios of responses to forcings, *Earth System Dynamics* 4, 31–49, doi.org/10.5194/esd-4-31-2013)

Let z be a climate state (temperature, sea-level, sea-ice, etc.).
Let g be the direct forcing of this state.
Let $\phi(z)$ be the instantaneous response of the state to the forcing.

$$\dot{z} = g(t) + \phi(z)$$

Pre-industrial equilibrium:

$$\dot{z} = g(t) + \phi(z) \stackrel{!}{=} 0 \Longrightarrow g^*(t) = -\phi(z^*)$$

Is this relation expected from theory?

Conceptualization of the response of a climate variable to forcing

Define

$$x := z - z^*, x_0 = 0, \qquad \text{change in state from PI}$$
$$f := g - g^*, f_0 = 0, \qquad \text{anthropogenic forcing}$$

$$\Longrightarrow \dot{x} = f(t) + \phi(x)$$

Linearize in (g^*, z^*):

$$\dot{x} \doteq f(t) - kx$$

Is this relation expected from theory?

Conceptualization of the response of a climate variable to forcing

Assume that anthropogenic forcing has exponential shape

$$f(t) = f_0 e^{rt}.$$

Then, it can be shown that the state follows

$$x(t) = \frac{f_0}{r+k} e^{rt}.$$

Ratios of responses to forcings:

$$\frac{x(t)}{f(t)} = \frac{1}{r+k}, \quad \frac{\dot{x}(t)}{f(t)} = \frac{r}{r+k}, \quad \frac{x(t)}{\int f(t)dt} = r$$

Example: The airborne fraction

Let C be atmospheric CO_2 concentrations.
Let $S = S_{LND} + S_{OCN}$ be CO_2 uptake from land and ocean.
Let $E(t) = E_0 e^{rt}$ be anthropogenic CO_2 emissions.

$$\dot{C} = E - S_{LND} - S_{OCN}, \text{ global carbon budget}$$

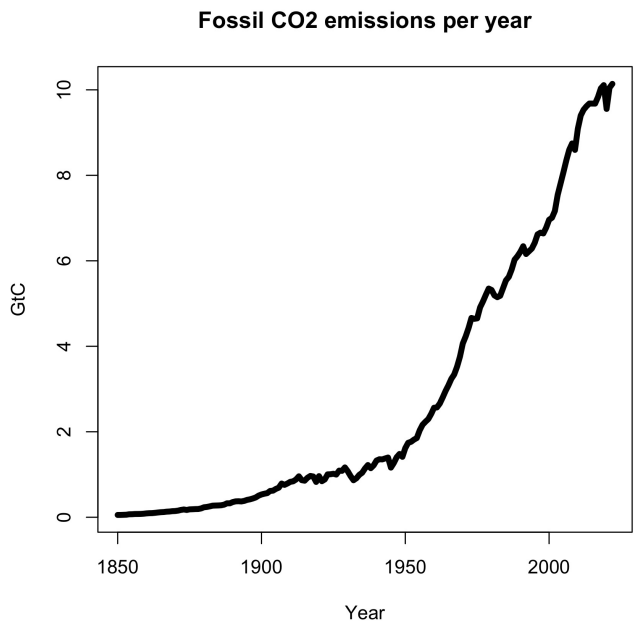
$$S = S_{LND} + S_{OCN} \doteq k\,C, \text{ sinks linear in concentrations}$$

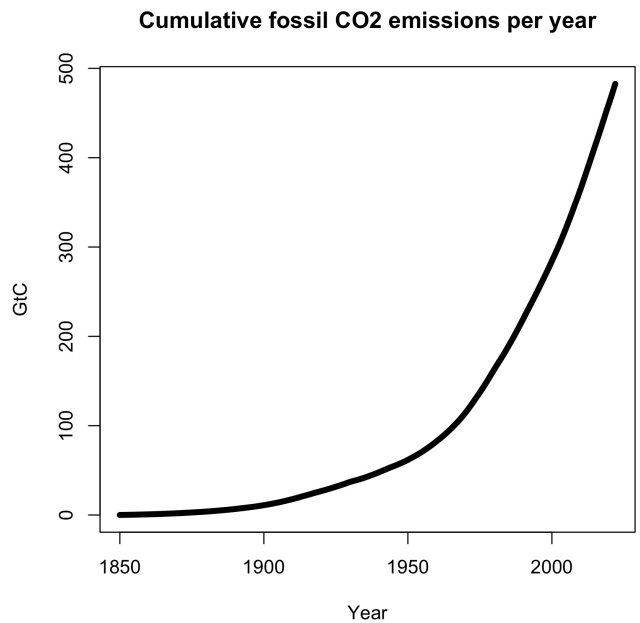
$$\Longrightarrow \dot{C} = E_0 e^{rt} - k\,C$$

$$\Longrightarrow C(t) = \frac{E_0}{r+k} e^{rt}$$

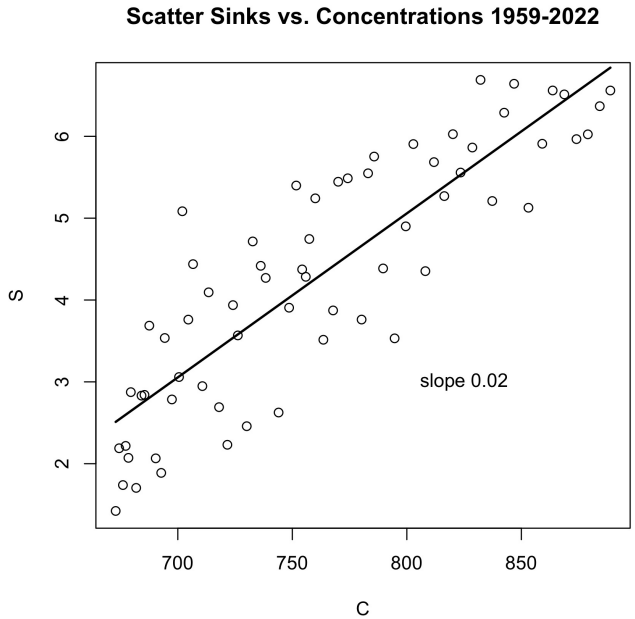
$$\frac{\dot{C}}{E} = \frac{r}{r+k} \text{ airborne fraction}$$

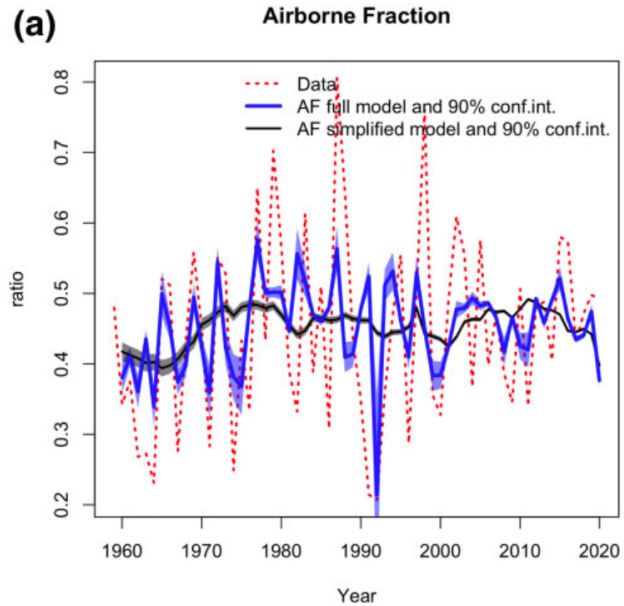
Example: The airborne fraction





Example: The airborne fraction





Right panel from M. Bennedsen, E. Hillebrand, S.J. Koopman, 2023, A multivariate statistical model of the global carbon budget 1959–2020, *Journal of the Royal Statistical Society Series A* 186(1), 20–42

$\hat{r}$ on 1959-2022 data: 0.016

$$\frac{r}{r+k} \approx \frac{0.016}{0.016+0.020} = \frac{4}{9} = 0.44444\ldots$$

Example: Energy balance

Incoming solar radiation: $S\pi r^2(1-a)$

S solar constant $\sim$ 1370 W/m^2
a albedo $\sim$ 0.30

Outgoing infrared radiation: $4\pi r^2 \epsilon \sigma T^4$ Stefan-Boltzmann law

Josef Stefan (1835-1893, born in Klagenfurt St. Peter, Ebentaler Str. 88, Prof (and Rektor) at U Wien)
Ludwig Eduard Boltzmann (1844-1906, born in Vienna, Prof at U Graz, U Wien, U München, U Leipzig)

ϵ emissivity, (1-infrared albedo)
σ Stefan-Boltzmann constant $\sim$ 5.67e-8
T Earth's effective emission temperature $\sim$ 288K

Example: Energy balance

$$
\begin{aligned}
\frac{S}{4}(1-a) &= \epsilon\sigma T^4, \\
&= \epsilon\sigma(T_0 + T_C)^4 \approx \epsilon\sigma(273+15)^4 \\
&= \epsilon\sigma T_0^4 \left(1 + 4\frac{T_c}{T_0} + 6(\frac{T_c}{T_0})^2 + 4(\frac{T_c}{T_0})^3 + (\frac{T_c}{T_0})^4\right), \\
&\doteq \underbrace{\epsilon\sigma T_0^4}_{A} + \underbrace{4\epsilon\sigma T_0^3}_{B} T_c
\end{aligned}
$$

Example: Energy balance

Energy imbalance drives changes in global temperatures, mediated by a heat capacity C of the planet.

$$C\frac{dT}{dt} = \underbrace{Q(1-a)}_{\text{forcing}} - \underbrace{\epsilon\sigma T^4}_{\text{feedback}}, \quad Q = S/4,$$
$$\doteq Q(1-a) - A - B\,T_c.$$

Pre-industrial:

$$\underbrace{Q(1-a^*)}_{F^*} - \epsilon^*\sigma(T^*)^4 = 0.$$

Change variables: $F = F - F^*$ anthropogenic forcing,
$T = T - T^*$ post-industrial temperatures

Example: Energy balance

Standard linear energy balance model with single layer

$$C\frac{dT}{dt} = F - \lambda T$$

Standard linear energy balance model with two layers

$$C_m\frac{dT_m}{dt} = F - \lambda T_m - \gamma(T_m - T_d),$$
$$C_d\frac{dT_d}{dt} = \gamma(T_m - T_d)$$

Other examples

- **Sea level**: S. Rahmstorf, 2007, A Semi-Empirical Approach to Projecting Future Sea-Level Rise, *Science* 315, 368, doi.org/10.1126/science.1135456
- **Sea ice**: D. Notz and J. Stroeve, 2016, Observed Arctic sea-ice loss directly follows anthropogenic CO_2 emission, *Science* 354, 6313, doi.org/10.1126/science.aag2345

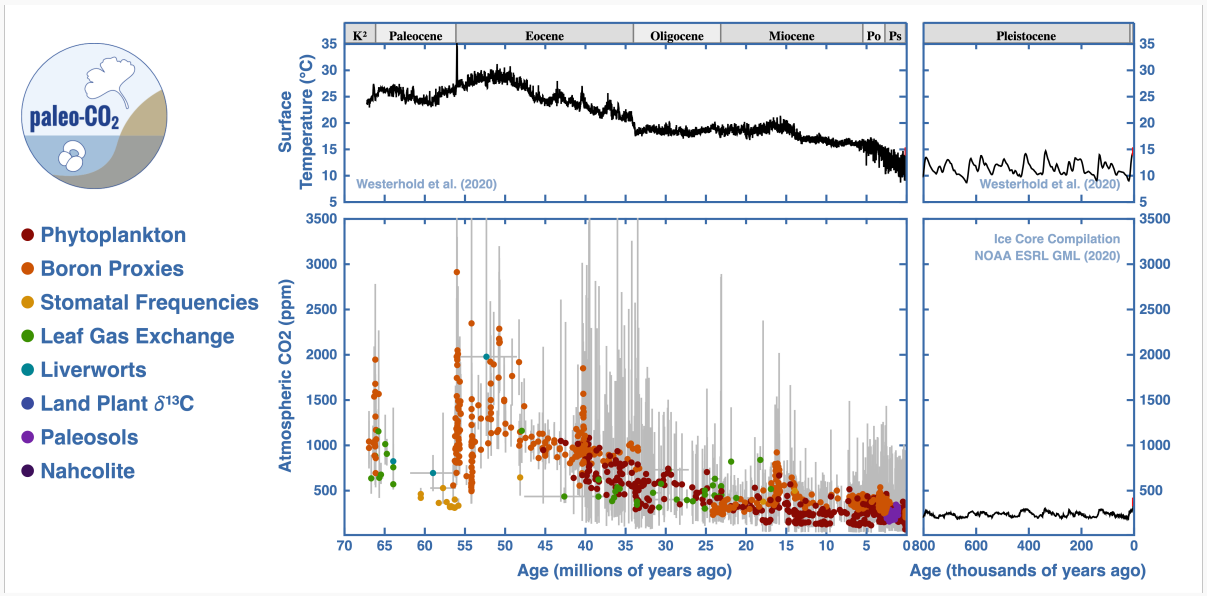
3. The CO_2–climate record in deep time

The CO_2–climate record in deep time

From modern linearity to deep time

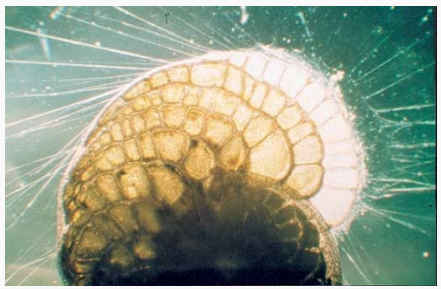
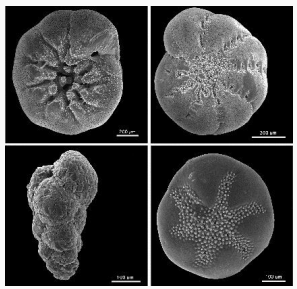
- The near-linear warming/cumulative-emissions relation is an instrumental-era regularity.
- Across 70 million years the CO_2–climate link is state-dependent: thresholds, hysteresis, orbital pacing.

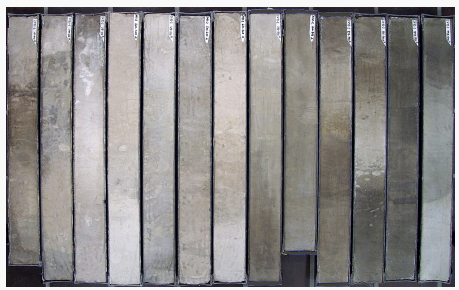
Question: where does today's 425 ppm sit relative to the CO_2 levels at which the great ice sheets formed?



The CO_2–climate record in deep time

Foraminifera

Living foraminifer; fossil benthic shell; drilling vessel; ocean sediment cores; core repository.

The CO_2–climate record in deep time

The shell is geological gold

- Foraminifera build shells of $CaCO_3$ from oxygen and carbon in seawater.
- Oxygen occurs as ^{16}O and ^{18}O; carbon as ^{12}C and ^{13}C.
- In cold water, shells take up relatively more ^{18}O.

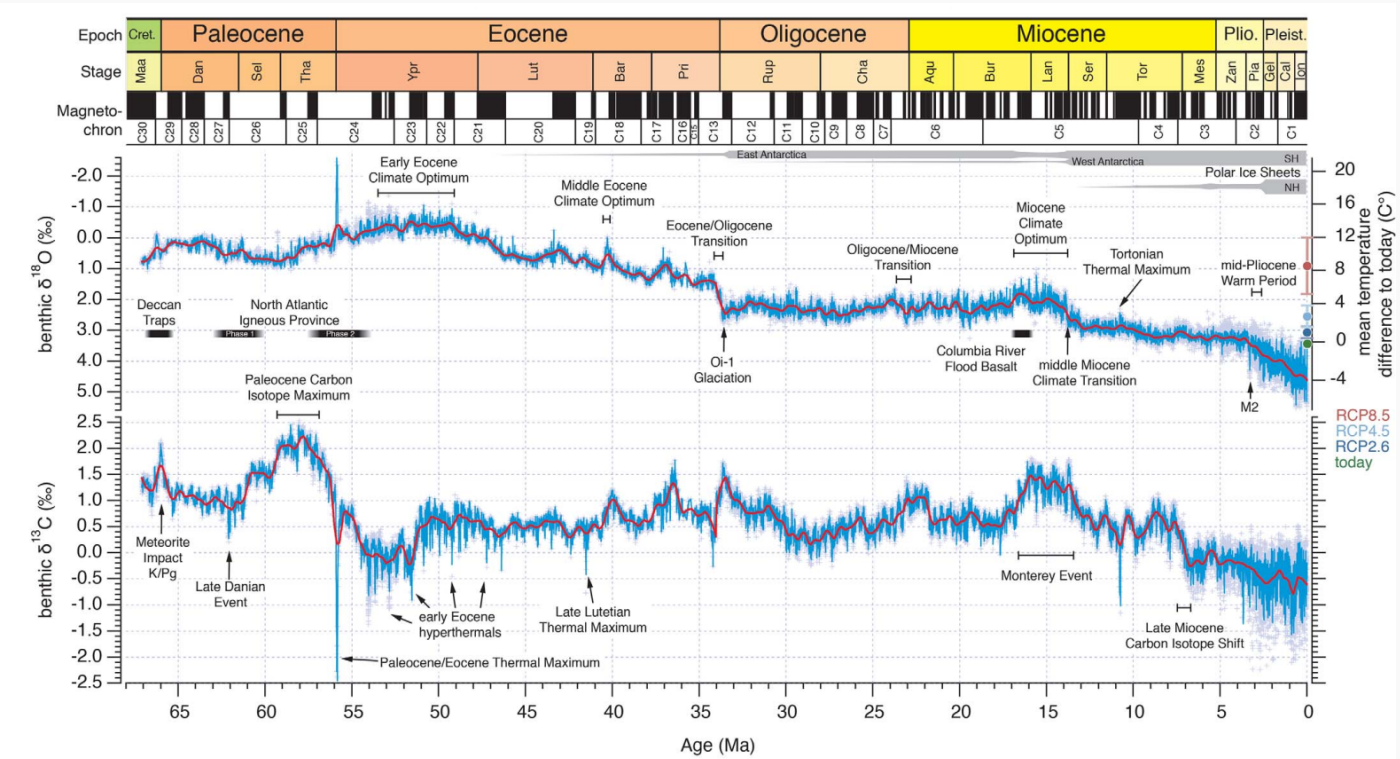
$$\delta^{18}O = \left(\frac{(^{18}O/^{16}O)_{\text{sample}}}{(^{18}O/^{16}O)_{\text{standard}}} - 1 \right) \times 1000 \text{ (per mil)}$$

- Benthic $\delta^{18}O$: paleothermometer (temperature and ice volume); $\delta^{13}C$: carbon cycle.
- Planktic shells also incorporate boron, from which pCO_2 is inferred.

The CO_2–climate record in deep time

Data: $\delta^{18}O$ and $\delta^{13}C$

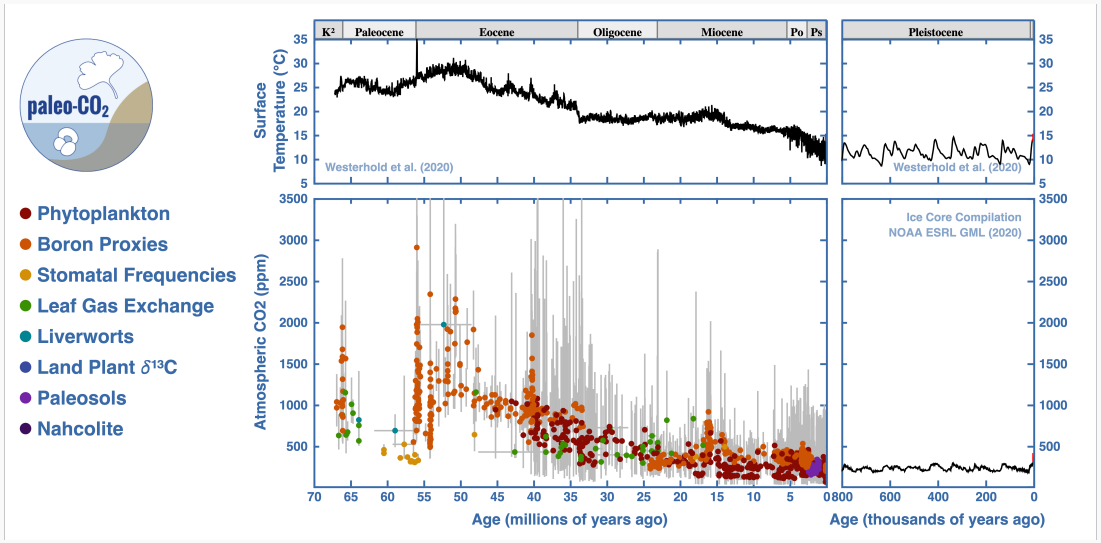
CENOGRID: 10 drill sites, 34 studies, 23 species. Westerhold et al. (2020).



The CO_2–climate record in deep time

Data: atmospheric CO_2

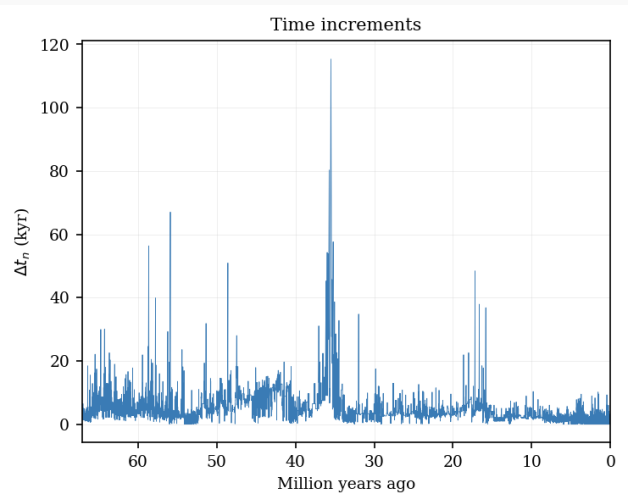
Multi-proxy compilation (CenCO2PIP), 13 proxy categories. paleo-co2.org.

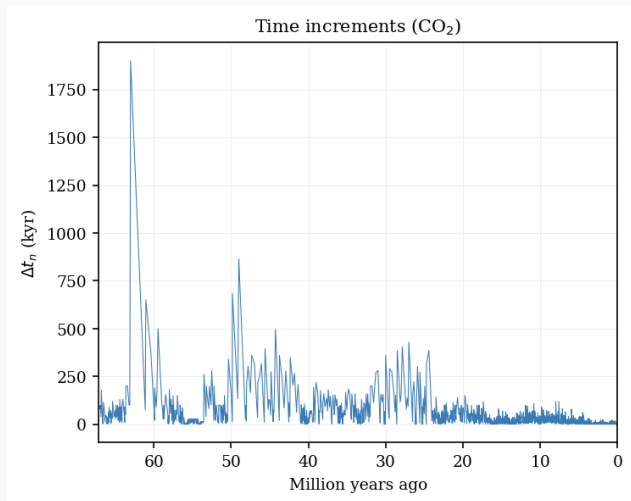


The CO_2–climate record in deep time

Awkward features for time-series models

Tens of Myr; irregular and near-simultaneous time stamps; sparse early, dense late; non-stationary with temporary trends.





Left: $\delta^{18}O$ and $\delta^{13}C$ sampling increments. Right: CO_2 sampling increments.

The CO_2–climate record in deep time

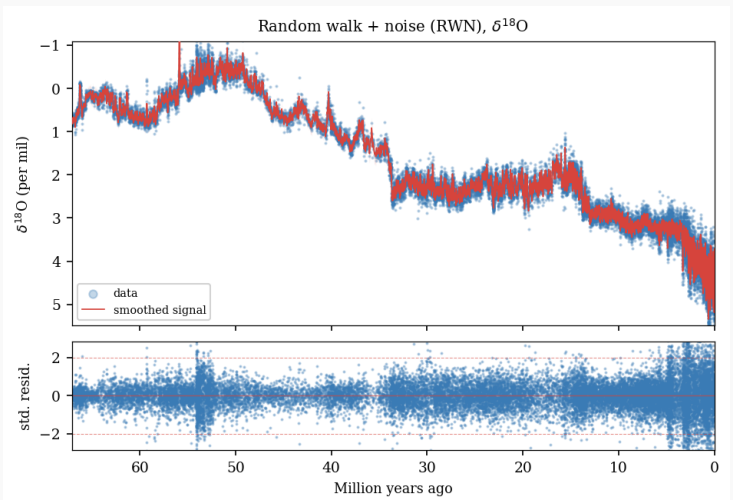
Random walk + noise (RWN), univariate $\delta^{18}O$

Bennedsen, Hillebrand, Koopman, Larsen, Lupien (2025).

Simplest case: one measurement variance, one state variance; no sites, no periods.

$$y_n = \mu_{t_n} + \varepsilon_n, \qquad \varepsilon_n \sim N(0, \sigma_\varepsilon^2),$$
$$\mu_{t_n+\Delta t_n} = \mu_{t_n} + \eta_{t_n}, \qquad \eta_{t_n} \sim N(0, \sigma_\eta^2 \, \Delta t_n).$$

State variance scales with the gap Δt_n (Brownian motion); μ extracted by maximum likelihood and the Kalman smoother.



The CO_2–climate record in deep time

Integrated random walk + noise, IWN(m)

Bennedsen, Hillebrand, Koopman, Larsen, Lupien (2025).

Level is the integral of a slope; exact continuous-time discretization over Δt_n. For $m = 2$:

$$\begin{bmatrix} \mu \\ \mu^{(1)} \end{bmatrix}_{t_n+\Delta t_n} = \begin{bmatrix} 1 & \Delta t_n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mu \\ \mu^{(1)} \end{bmatrix}_{t_n} + \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}.$$

General order m: $T_{jk} = \Delta t^{\,k-j}/(k-j)!$. For $m = 4$ the transition is

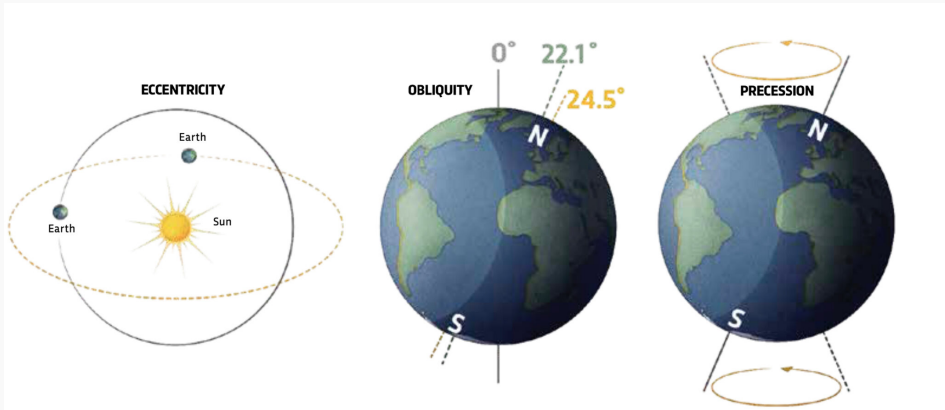
$$T(\Delta t) = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} & \frac{\Delta t^3}{6} \\ 0 & 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Discrete time is the special case $\Delta t = 1$.

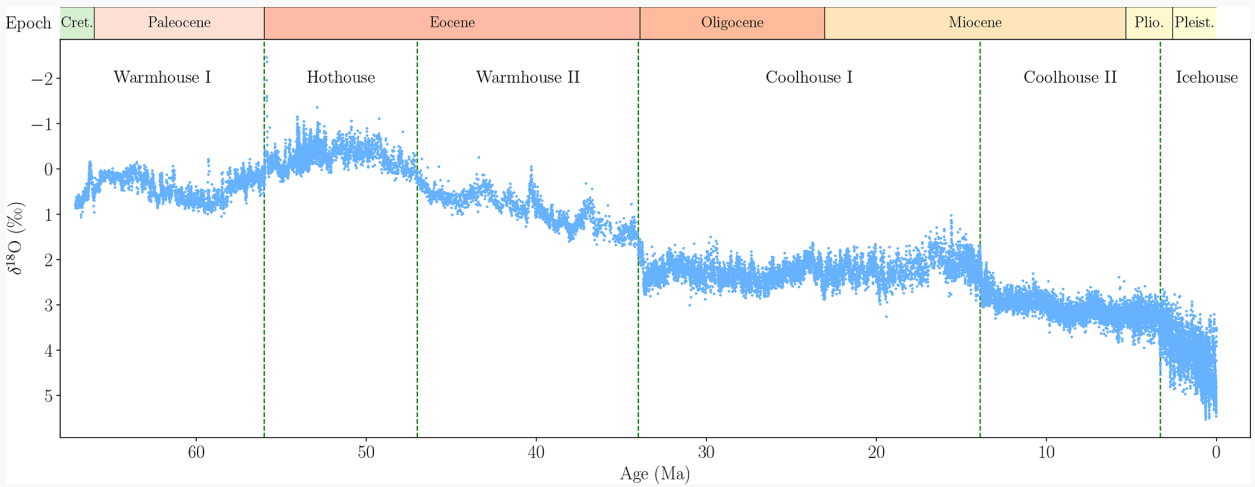
The CO_2–climate record in deep time

Milankovitch forcing and climate epochs

Orbital solution La2004, Laskar et al. (2004). Climate states: Westerhold et al. (2020).



eccentricity (∼100, 405 kyr), obliquity (∼41 kyr), precession (∼21 kyr)



Six Cenozoic climate states (Warmhouse, Hothouse, Coolhouse, Icehouse)

The CO_2–climate record in deep time

State (transition) equations

Bennedsen, Hillebrand, Koopman, Larsen, Lupien (2025).

Three coupled levels, $p \in \{\delta^{18}\mathrm{O},\ \delta^{13}\mathrm{C},\ \log \mathrm{CO}_2\}$, with state-$k$ variances, correlations, and orbital forcing:

$$\mu^{(p)}_{t_n+\Delta t_n} = \mu^{(p)}_{t_n} + \left(B_k\, \Delta o_{t_n}\right)_p + \eta^{(p)}_{t_n}, \qquad \boldsymbol{\eta}_{t_n} \sim N(0,\ Q_k\, \Delta t_n),$$

$$Q_k = \begin{bmatrix} \sigma^2_{1k} & \rho_{12,k}\sigma_{1k}\sigma_{2k} & \rho_{13,k}\sigma_{1k}\sigma_{3k} \\ \cdot & \sigma^2_{2k} & \rho_{23,k}\sigma_{2k}\sigma_{3k} \\ \cdot & \cdot & \sigma^2_{3k} \end{bmatrix}.$$

Orbital increments $\Delta o = (\Delta e, \Delta\varepsilon, \Delta\varpi)$; loadings B_k and covariance Q_k vary across the six Westerhold states.

The CO_2–climate record in deep time

Measurement equations

$N = 54{,}498$ observations: 24,259 $\delta^{18}O$, 23,939 $\delta^{13}C$, 6,300 log CO_2.

One scalar update per observation (one proxy at a time):

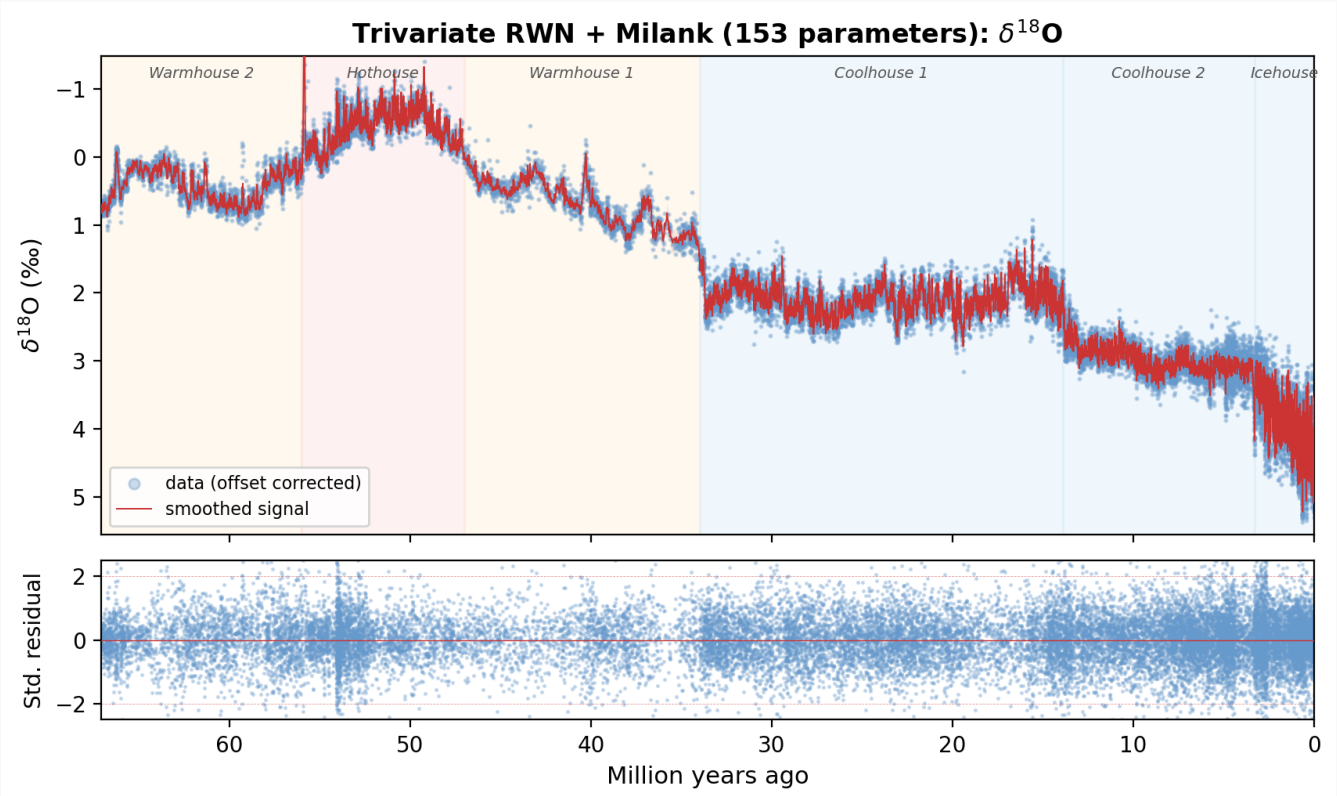
$$
\begin{aligned}
y_n^{\delta^{18}\mathrm{O}} &= \mu_{t_n}^{\delta^{18}\mathrm{O}} + c_s + \varepsilon_n, \qquad \varepsilon_n \sim N(0, \sigma^2_{\varepsilon,s}),\\
y_n^{\delta^{13}\mathrm{C}} &= \mu_{t_n}^{\delta^{13}\mathrm{C}} + c_s + \varepsilon_n,\\
y_n^{\log \mathrm{CO}_2} &= \mu_{t_n}^{\log \mathrm{CO}_2} + c_c + \varepsilon_n.
\end{aligned}
$$

Bias intercepts place all sources on one latent scale: drill site c_s, $s \in \{1, \ldots, 10\}$ (for $\delta^{18}O$ and $\delta^{13}C$); proxy category c_c, $c \in \{1, \ldots, 13\}$ (for CO_2). One reference source is fixed for identification.

The CO_2–climate record in deep time

The extracted $\delta^{18}O$ signal

Trivariate RWN with period-dependent Milankovitch forcing. Bennedsen, Hillebrand, Koopman, Larsen, Lupien (2025).

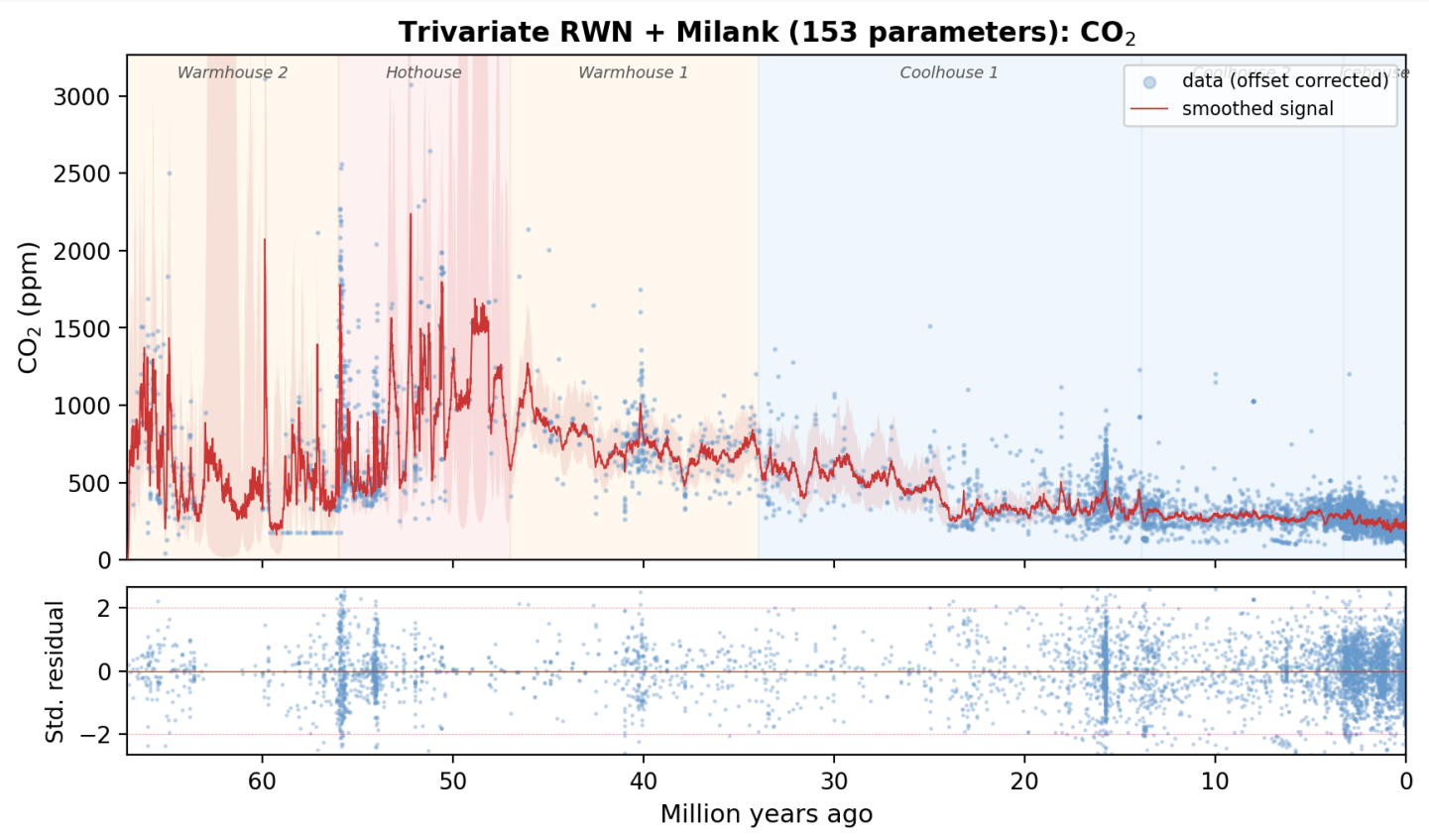


Smoothed state (red) threads the multi-site data cloud (blue); band = 95%. Axis inverted (down = colder).

The CO_2–climate record in deep time

The extracted CO_2 signal

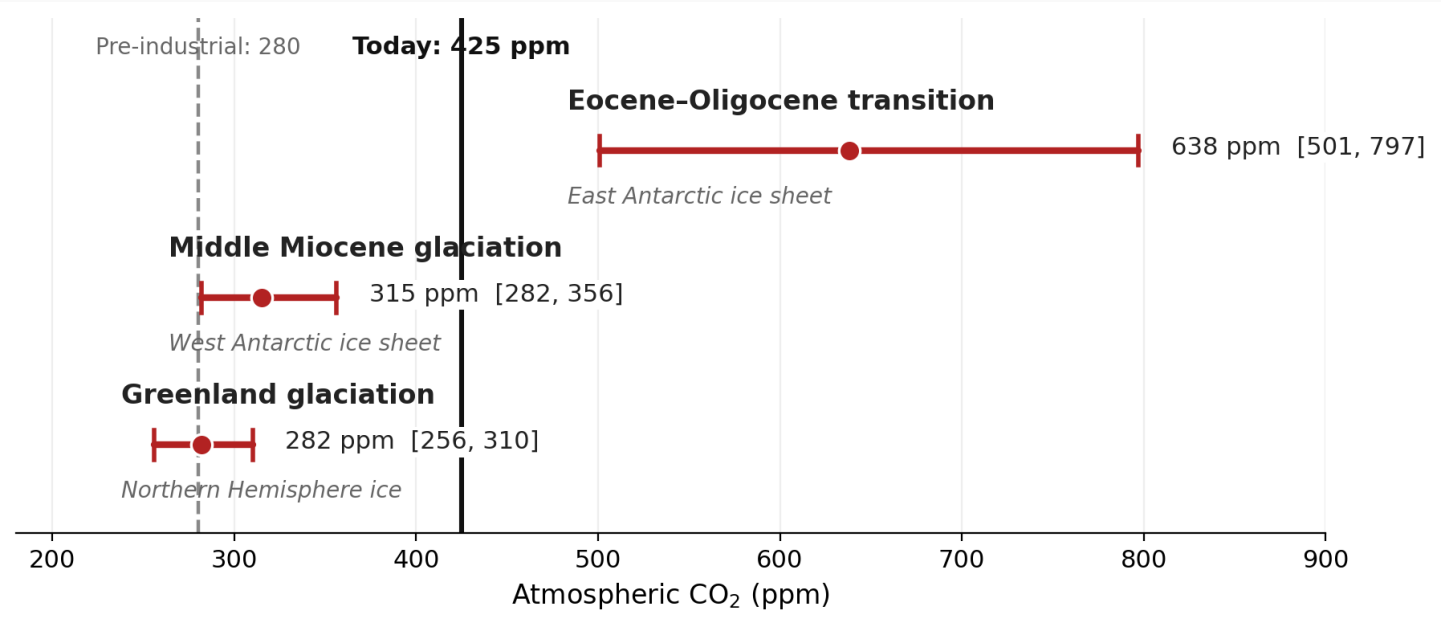
Trivariate RWN with period-dependent Milankovitch forcing. Bennedsen, Hillebrand, Koopman, Larsen, Lupien (2025).



The CO_2–climate record in deep time

The CO_2 thresholds of the great glaciations

Per-window median CO_2 and 95% band, trivariate model. Bennedsen, Hillebrand, Koopman, Larsen, Lupien (2025).



The CO_2–climate record in deep time

Conclusions

- Continuous-time state space extracts latent climate signals with calibrated uncertainty from irregular, multi-site, multi-proxy data over the last 70 million years.
- Milankovitch forcing exerts a stronger influence in cold regimes, weaker in warm Earths.
- Today's 425 ppm sits above the Greenland and West Antarctic glaciation thresholds, below the East Antarctic threshold.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