The VAR model

Suppose that $\boldsymbol{y}_t \in \mathbb{R}^m$, $t = 1, \ldots, T$, follows a zero-mean heteroskedastic VAR(p) process,

$$\boldsymbol{y}_t = \boldsymbol{A}_1 \boldsymbol{y}_{t-1} + \cdots + \boldsymbol{A}_p \boldsymbol{y}_{t-p} + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}_m(\boldsymbol{0}, \boldsymbol{\Omega}_t)$$

- $\boldsymbol{A}_j$ ($j = 1, \ldots, p$): $m \times m$ matrices of autoregressive coefficients

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- $\boldsymbol{x}_t = (\boldsymbol{y}'_{t-1}, \ldots, \boldsymbol{y}'_{t-p})'$ and a $m \times mp$ coefficient matrix $\boldsymbol{B} = (\boldsymbol{A}_1, \ldots, \boldsymbol{A}_p)$ to rewrite the model more compactly as

$$\boldsymbol{y}_t = \boldsymbol{B}\boldsymbol{x}_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}_m(\boldsymbol{0}, \boldsymbol{\Omega}_t).$$

- Or equivalently

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{B}' + \boldsymbol{E}$$

where $\boldsymbol{Y} = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T)'$, $\boldsymbol{X} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T)'$ and $\boldsymbol{E} = (\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_T)'$

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(Including an intercept is straightforward but omitted here for simplicity of exposition)

Catering for stochastic volatility

Decompose the time-varying covariance matrix into

$$\boldsymbol{\Omega}_t = \boldsymbol{\Lambda} \boldsymbol{V}_t \boldsymbol{\Lambda} + \boldsymbol{\Sigma}_t$$

where

- $\boldsymbol{\Sigma}_t = \text{diag}(\sigma^2_{1t}, \ldots, \sigma^2_{mt})$ and $\boldsymbol{V}_t = \text{diag}(\sigma^2_{m+1,t}, \ldots, \sigma^2_{m+q,t})$
- $\boldsymbol{\Lambda}$: $m \times q$ factor loadings matrix
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Known as the factor stochastic volatility model (see e.g. Pitt and Shephard, 1999; Aguilar and West, 2000) with latent variable representation

$$\boldsymbol{\varepsilon}_t \sim \mathcal{N}_m(\boldsymbol{\Lambda} \boldsymbol{f}_t, \boldsymbol{\Sigma}_t), \quad \boldsymbol{f}_t \sim \mathcal{N}_q(\boldsymbol{0}, \boldsymbol{V}_t)$$

- Off-diagonal entries of $\boldsymbol{\Omega}_t$ exclusively stem from the volatilities of the q factors
- diagonal entries of $\boldsymbol{\Omega}_t$ are allowed to feature idiosyncratic deviations driven by the elements of $\boldsymbol{\Sigma}_t$
- reduces the number of free elements in $\boldsymbol{\Omega}_t$ from $m(m+1)/2$ to mq
- enables us to derive an efficient Gibbs sampler that allows for conditional equation-by-equation estimation

Shrinking the coefficients

Dirichlet-Laplace prior (Bhattacharya et al., 2015) for $\boldsymbol{b} = \text{vec}(\boldsymbol{B})$. In what follows, we impose the DL prior on each of the $K = m^2 p$ elements of $\boldsymbol{b}$, for $j = 1, \ldots, K$,

$$b_j \sim \mathcal{N}(0, \psi_j \vartheta_j^2 \zeta)$$

- ψ_j are local scaling parameters $\psi_j \sim \text{Exp}(1/2)$
- $\boldsymbol{\vartheta} = (\vartheta_1, \ldots, \vartheta_K)' \in \mathcal{S}^{K-1} = \{\boldsymbol{\vartheta} : \vartheta_j \geq 0, \sum_{j=1}^K \vartheta_j = 1\}$, $\boldsymbol{\vartheta} \sim \text{Dir}(a, \ldots, a)$
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Nice features:

- strong shrinkage with plenty of flexibility
- mimicking "real" spike and slab priors, but much lower computational burden
- implementation is simple, only requires one single structural hyperparameter a (smaller $a \Rightarrow$ stronger spike)
- good contraction properties in stylized models when $a = K^{-(1+\epsilon)}$ with ϵ small

Other priors

Factor loadings: iid normal $\lambda_{ij} \sim \mathcal{N}(0, 1)$ for $i = 1, \ldots, m$ and $j = 1, \ldots, q$ (additional shrinkage possible as in Kastner, forthcoming).

Log volatilities:

- Level: $\mu_k \sim \mathcal{N}(0, M_\mu)$, $k = 1, \ldots, m$
- Persistences: $(\phi_k + 1)/2 \sim \mathcal{B}(a_0, b_0)$, $k = 1, \ldots, m + q$, to ensure stationarity (cf. Kim et al., 1998)
- Volatility of volatility: $\varsigma_k^2 \sim \mathcal{G}(\frac{1}{2}, \frac{1}{2\xi})$, $k = 1, \ldots, m + q$ (cf. Frühwirth-Schnatter and Wagner, 2010)

MCMC at a glance

- Local shrinkage parameters $\psi_j|\bullet \sim iG(\vartheta_j\zeta/|b_j|, 1),\ j = 1, \ldots, K$ via efficient and stable rejection sampler from Hörmann and Leydold (2013), R-package `GIGrvg`
- Global shrinkage parameter $\zeta|\bullet \sim \mathcal{GIG}\left(K(a-1), 1, 2\sum_{j=1}^{K}|b_j|/\vartheta_j\right)$, `GIGrvg`
- Scaling parameters ϑ_j by first sampling L_j from $L_j|\bullet \sim \mathcal{GIG}(a-1, 1, 2|b_j|)$, and then setting $\vartheta_j = L_j/\sum_{i=1}^{K} L_i$ (Bhattacharya et al., 2015), `GIGrvg`
- Conditionally "univariate" SV states and parameters of volatility processes via auxiliary mixture sampling (Omori et al., 2007) with ASIS (Yu and Meng, 2011) via R-package `stochvol`
- Factors and factor loadings: "deep interweaving" (Kastner et al., 2017) via R-package `factorstochvol`
- VAR parameters, see next slide

Sampling the VAR parameters

Exploiting the data augmentation representation for the factor SV model, the model can be cast as a system of m conditionally unrelated regression models

$$z_{it} := y_{it} - \boldsymbol{\Lambda}_{i\bullet}\boldsymbol{f}_t = \boldsymbol{B}_{i\bullet}\boldsymbol{x}_t + \eta_{it}, \quad i = 1, \ldots, m, \quad t = 1, \ldots, T$$

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Full conditional posteriors:

$$\boldsymbol{B}_{i\bullet}'|\bullet \sim \mathcal{N}(\boldsymbol{b}_i, \boldsymbol{Q}_i), \quad \boldsymbol{Q}_i = (\tilde{\boldsymbol{X}}_i'\tilde{\boldsymbol{X}}_i + \boldsymbol{\Phi}_i^{-1})^{-1}, \quad \boldsymbol{b}_i = \boldsymbol{Q}_i(\tilde{\boldsymbol{X}}_i'\tilde{\boldsymbol{z}}_i)$$

- $\boldsymbol{\Phi}_i$ is the respective $k \times k$ diagonal submatrix of $\boldsymbol{\Phi} = \zeta \times \text{diag}(\psi_1\vartheta_1^2, \ldots, \psi_K\vartheta_K^2)$
- $\tilde{\boldsymbol{X}}_i$ is a $T \times k$ matrix with typical row t given by $\boldsymbol{X}_t/\sigma_{it}$
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$\Rightarrow$ allows for equation by equation estimation: each draw costs $O(m^4p^3 + Tm^3p^2)$ – as opposed to $O(m^6p^3)$ in the naive approach (cf. Carriero et al., 2015)

Yet another approach

In typical macro data $T \lesssim 200$ ... even faster sampling is possible via an algorithm proposed by Bhattacharya et al. (2016) for univariate regressions, applied to each equation:

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1. Sample independently $\boldsymbol{u}_i \sim \mathcal{N}(\boldsymbol{0}_k, \boldsymbol{\Phi}_i)$ and $\boldsymbol{\delta}_i \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I}_T)$
2. Use $\boldsymbol{u}_i$ and $\boldsymbol{\delta}_i$ to construct $\boldsymbol{v}_i = \tilde{\boldsymbol{X}}_i \boldsymbol{u}_i + \boldsymbol{\delta}_i$
3. Solve $(\tilde{\boldsymbol{X}}_i \boldsymbol{\Phi}_i \tilde{\boldsymbol{X}}_i' + \boldsymbol{I}_T)\boldsymbol{w}_i = (\tilde{\boldsymbol{z}}_i - \boldsymbol{v}_i)$ for $\boldsymbol{w}_i$
4. Set $\boldsymbol{B}_{i\bullet}' = \boldsymbol{u}_i + \boldsymbol{\Phi}_i \tilde{\boldsymbol{X}}_i' \boldsymbol{w}_i$

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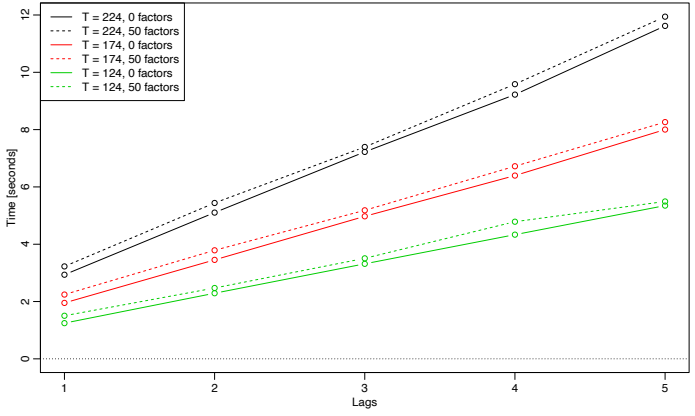
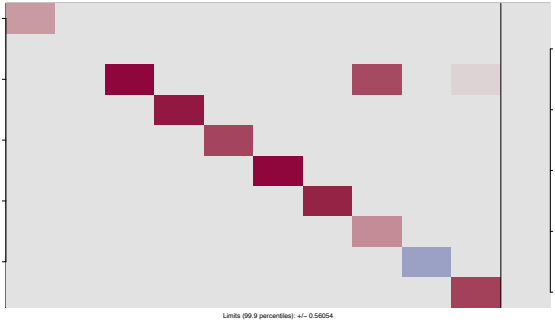
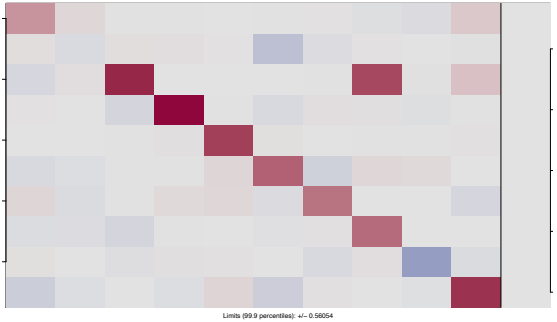
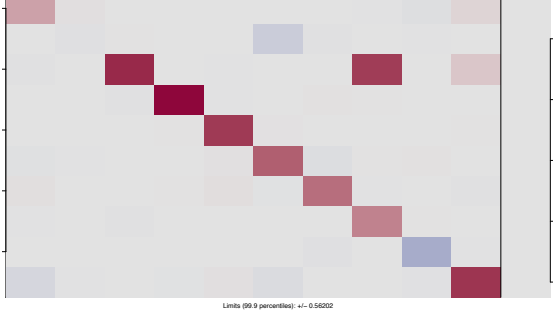
Cost is now $O(m^2 T^2 p)$,
e.g. for $m = 215$ we have:

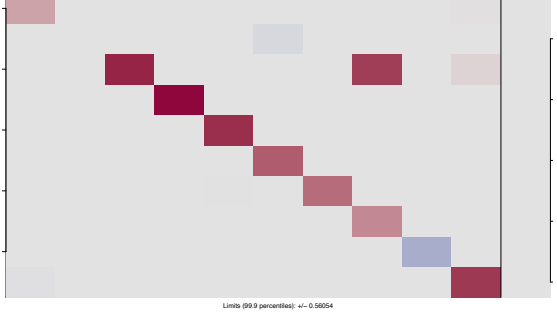


Simulation study ($T = 250, m = 10$): DGP, OLS, DL(1/2), DL(1/K)

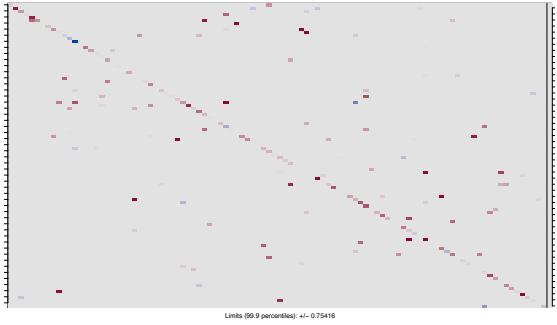


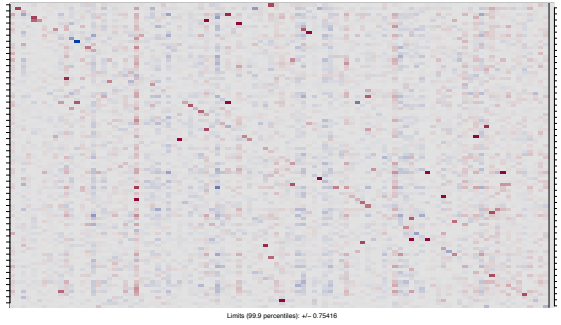


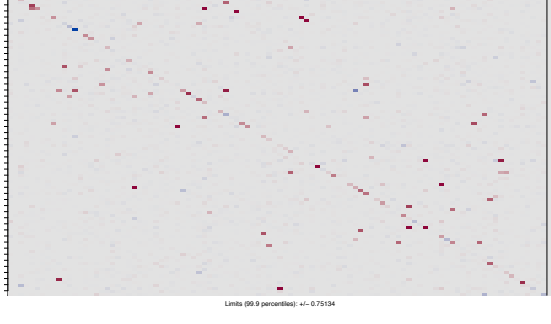


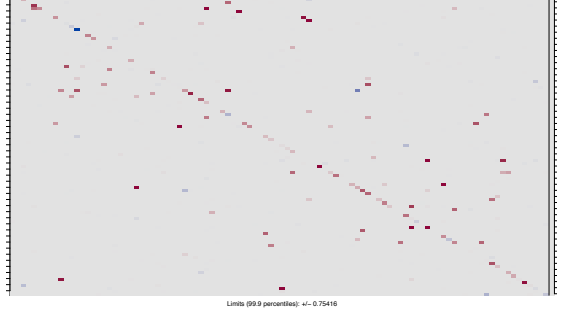


Simulation study ($T = 250, m = 100$): DGP, OLS, DL(1/2), DL(1/K)







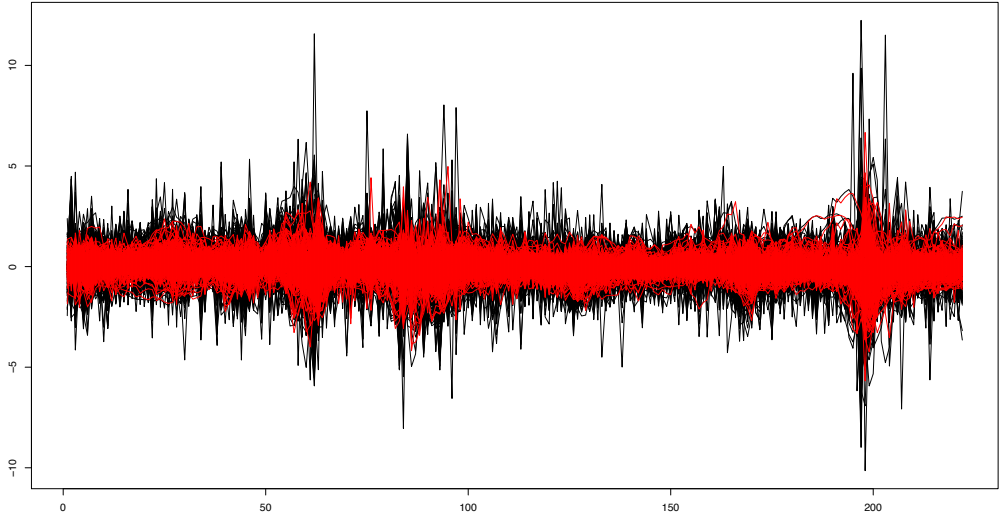


T \ m	sparse				intermediate				dense			
	10	20	50	100	10	20	50	100	10	20	50	100
DL ($a_{DL} = 1/2$)												
50	1.339	1.810	2.312	2.652	1.208	1.616	1.808	1.989	1.395	1.936	2.657	3.141
100	1.308	1.671	2.106	2.549	1.145	1.326	1.485	1.647	1.325	1.695	2.182	2.669
150	1.240	1.649	1.994	2.455	1.055	1.293	1.429	1.491	1.253	1.703	2.079	2.359
200	1.270	1.593	2.024	2.346	1.072	1.212	1.313	1.429	1.285	1.534	1.879	2.182
250	1.190	1.599	2.063	2.401	1.069	1.176	1.295	1.388	1.224	1.559	1.839	2.073
Normal-Gamma ($a_{NG} = 0.1$)												
50	1.100	1.448	1.949	2.701	0.985	1.314	1.560	2.037	1.073	1.496	2.203	3.208
100	1.057	1.296	1.726	2.157	1.006	1.133	1.301	1.483	1.091	1.327	1.806	2.271
150	1.030	1.348	1.645	2.042	0.934	1.147	1.268	1.318	1.070	1.423	1.722	1.988
200	1.058	1.301	1.653	1.940	0.923	1.064	1.168	1.260	1.023	1.282	1.569	1.860
250	0.969	1.322	1.680	1.987	0.945	1.029	1.146	1.231	1.056	1.288	1.548	1.754
Minnesota ($a_M = 0.0001$)												
50	1.142	1.186	1.309	1.383	1.062	1.035	1.022	0.996	1.125	0.892	0.863	0.743
100	1.466	1.492	1.760	1.963	1.307	1.151	1.188	1.186	1.382	1.093	1.043	0.966
150	1.764	1.731	1.992	2.328	1.526	1.272	1.331	1.304	1.601	1.230	1.226	1.036
200	1.909	1.909	2.265	2.591	1.596	1.381	1.374	1.429	1.692	1.297	1.250	1.138
250	1.825	2.122	2.548	2.930	1.625	1.413	1.478	1.527	1.689	1.464	1.366	1.188

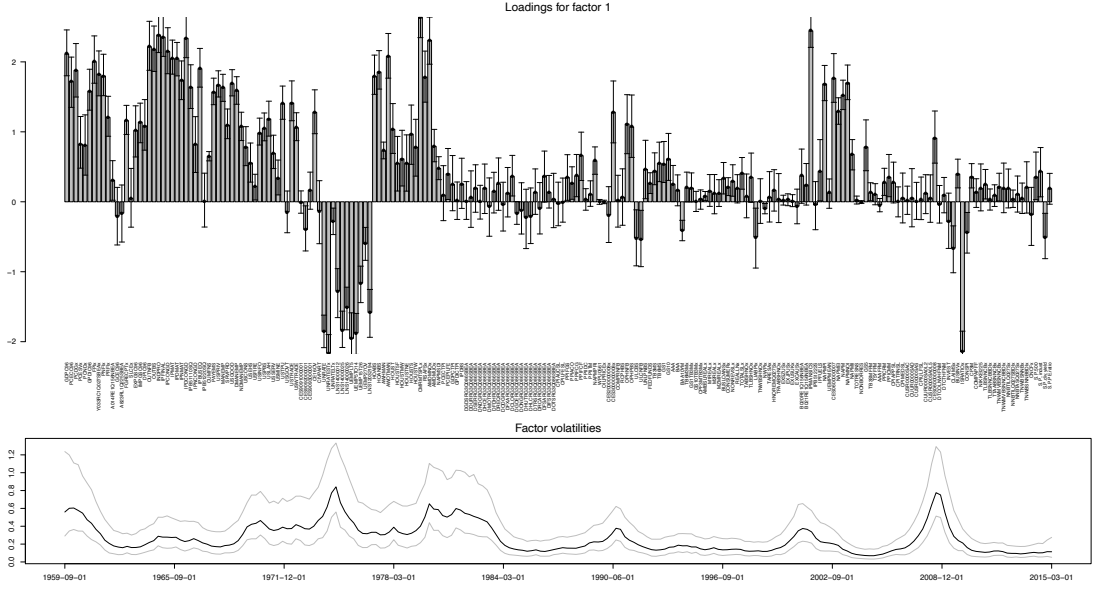
Modeling the US economy

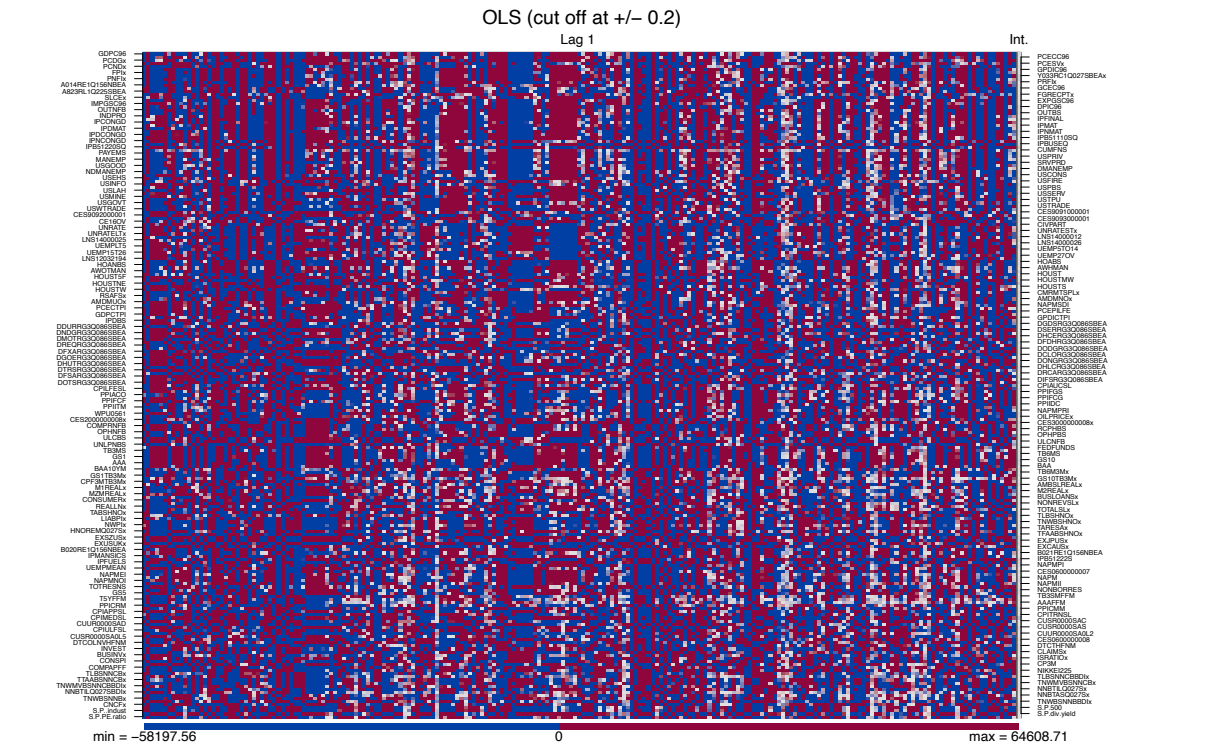
Quarterly dataset from McCracken and Ng (2016) including suggested transformations

- Ranging from 1959:Q1 to 2015:Q4, $m = 215$
- Component-wise standardization (zero mean, variance one)
- One lag, one factor (more on that later), $M_\mu = 10$, $a_0 = 20$, $b_0 = 1.5$, $\xi = 1$



Factor loadings and factor volatilities

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A823RL1Q225SBEA
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IMFSCSC96
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IPDCONGD
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IPBUSEQ
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MANEMP
USGOOD
NDMANEMP
USEHS
USINFO
USLAH
USMINE
USCONS
USWTRADE
CES9093000001
CE16OV
UNRATE
UNRATELTx
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UEMP5TO14
LNS14000026
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AWOTMAN
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PCECTPI
GDPCTPI
PCEPILFE
PCEPI
GPDICTPI
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DMOTRG3Q086SBEA
DHCRG3Q086SBEA
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CPINFB
OILPRICEx
ULCBS
MCOILWTICO
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MCOILBRENTEU
TB3MS
TBMMS
GS1
GS10
TB6MS
BAA
BAA10YM
TB6SMFFM
GS1TB3Mx
TB6MMx
GS10TB3Mx
AAAFFM
BAAFFM
MTREALx
AMBSLREALx
MZMREALx
BOGMBASEREALx
CONSUMERx
M1REALx
REALLNx
M2REALx
TOTALSLx
TB6SMFFMx
TB3SMFFMx
LIABPIx
TLBSHNOx
TNWBSHNOx
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EXUSUKx
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IPFUELS
IPB51222S
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NAPMEI
NAPMPI
NAPMII
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NONBORRES
GS5
TB3SMFFM
TSYFFM
AAAFFM
PPICRM
BAAFFM
CPIAPPSL
CP3Mx
CMRMTSPLx
PMNEOSL
CPITRNSL
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CUUR0000SAL
CUURP0000SA0L2
CUUR0000SA0L5
CUSR0000SA0L5
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DTCOLNVHFNM
DTCTHFNM
INVEST
INVCQRMTSPL
BUSINVx
ISRATIOx
CONSPI
GPSAVEC96
COMPAPFF
GPSAVE
TLBSNNCB
CP3M
TLBSNNCBBDIx
NIKKEI225
TNWBSNNCBBDIx
TNWBSNNCBSDIx
NBBTLQ027SBDIx
NNBTLQ027Sx
TNWBSNNBVx
TNWBSNNBSDIx
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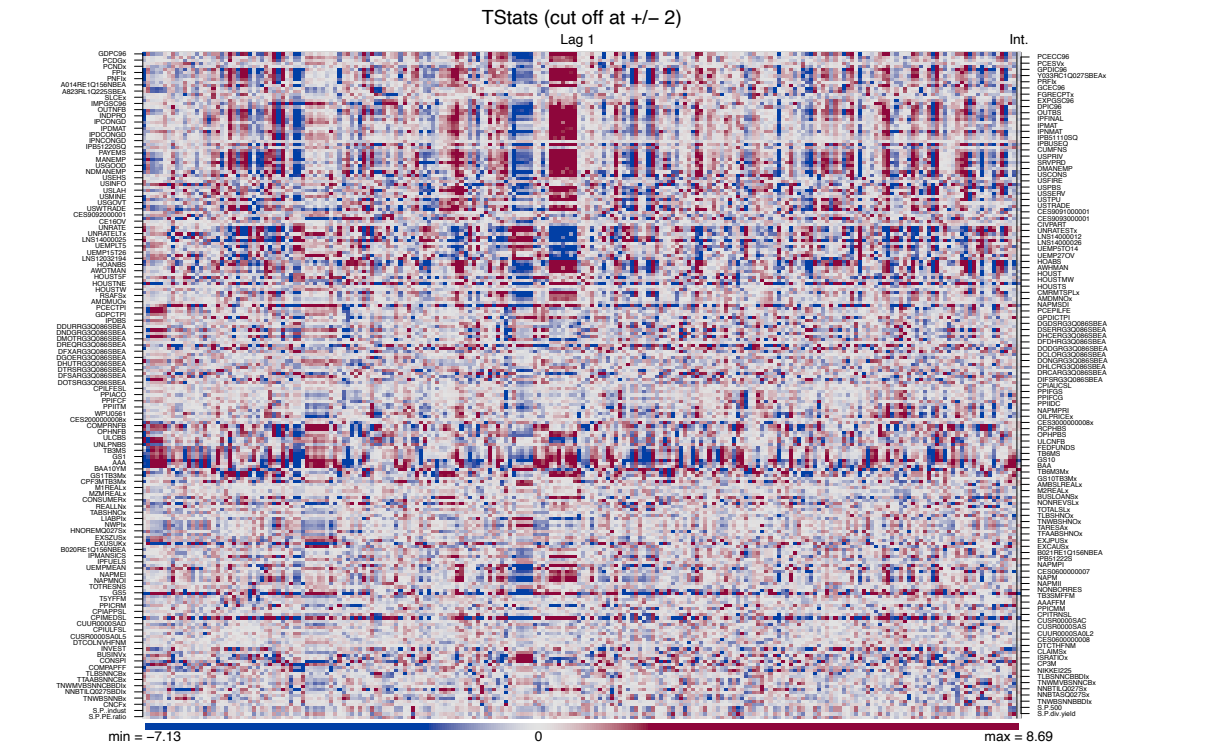
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INDPRO
IPCONGD
IPDMAT
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REALLNx
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EXUSUKx
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CONSPI
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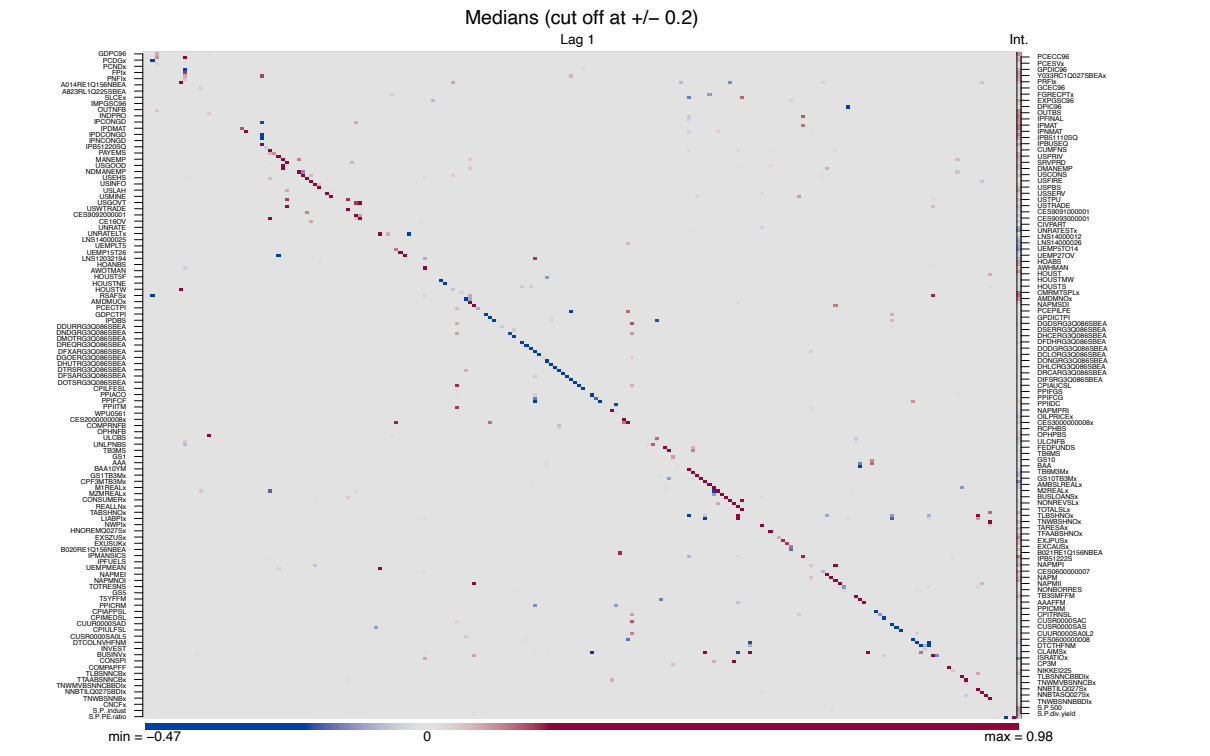
max = 8.69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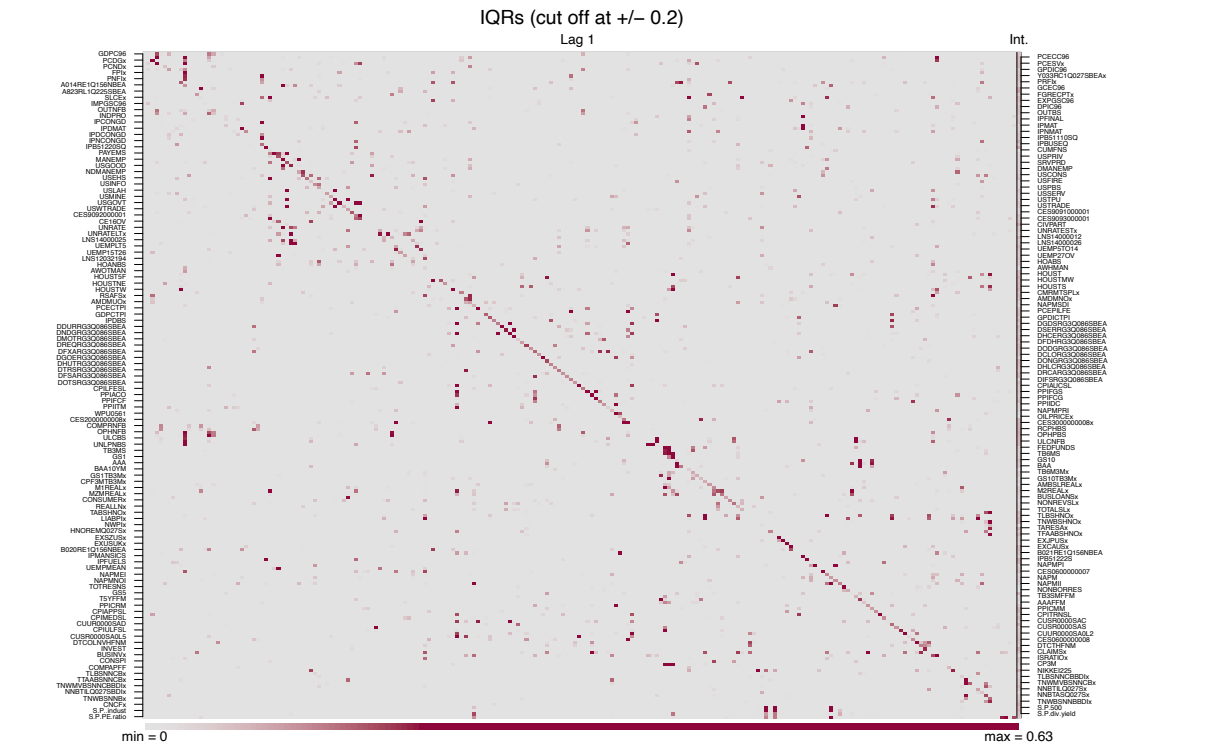
max = 0.98


IQRs (cut off at +/– 0.2)
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SCEAx
IMPGSC96
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INDPRO
IPCONGD
IPDMAT
IPDCONGD
IPNMAT
IPBUSEQ
PAYEMS
MANEMP
USGOOD
NDMANEMP
USCHS
USINFO
USLAH
USMINE
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USWTRADE
CES9093000001
CE16OV
UNRATE
UNRATELTx
LNS14000025
UEMPLT5
UEMP5TO14
LNS14000036
HOAMS
AWOTMAN
HOUSTSF
HOUSTNE
HOUSTW
RSAFSx
AMDMNOx
PCECTPI
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PDSBS
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DNDGRG3Q086SBEA
DMOTRG3Q086SBEA
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DFXARG3Q086SBEA
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COMPNFB
CPINFB
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TB3MS
GS1
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GS1TB3Mx
CPF3MTB3Mx
MTREALx
M2MREALx
CONSUMERx
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LABSHx
HNOREMQ027Sx
EXSZUSx
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NAPMEI
NAPMPRI
TOTRESNS
GS5
TSYFFM
PPICRM
CPIAPPSL
PIMEDSL
CUSR0000SA0L5
CPIULFSL
CUSR0000SA0L5
DTCOLNVHFNM
INVEST
BUSINVx
CONSPI
COMPNPFF
TLBSNNCBx
TTLBSNNCBx
TNWMVBSNNCBSx
NNBTLQ027SBDIx
TNWBSNNBx
CNCFx
S.P.indust
S.P.PE.ratio
PCECC96
PCESVx
GPDIC96
Y033RC1Q027SBEAx
PRFIx
GCEC96
FGRECPTx
EXPGSC96
DPIC96
CNP16OV
IPMAT
IPNMAT
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IPBUSEQ
CUMFNS
USPRIV
SRVPRD
DMANEMP
USGOOD
USPRE
USPBS
USSERV
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UEMP27OV
HOABS
AWHMAN
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HOUSTS
CMRMTSPLx
AMDMNOx
NAPMSDI
PCEPILFE
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DNDGRG3Q086SBEA
DHCRRG3Q086SBEA
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DONRRG3Q086SBEA
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DRCARG3Q086SBEA
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OPCPHS
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FEDFUNDS
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BAA
TB6MSMx
GS10TB3Mx
AMBSLREALx
M2REALx
BOGAMBSx
NONREVSLx
TOTALSLx
TLBSHNOx
TNWBSHNOx
TARESAx
TRARSHNOx
EXJPUSx
EXCHUSx
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TBSMFFM
AAAFFM
BAAFFM
CPITNSL
CUSR0000SAC
CUSR0000SAS
CUUR0000SA0L2
CES9091000008
DTCTHFNM
INVCQ
ISRATIOx
CP3M
NIKKEI225
TLBSNNCBBDIx
TNWMVBSNNCBSx
NNBTLQ027Sx
NNBTASQ027Sx
TNWBSNNBBDIx
S.P.500
S.P.div.yield
min = 0
max = 0.63

Out-of-sample predictive evidence

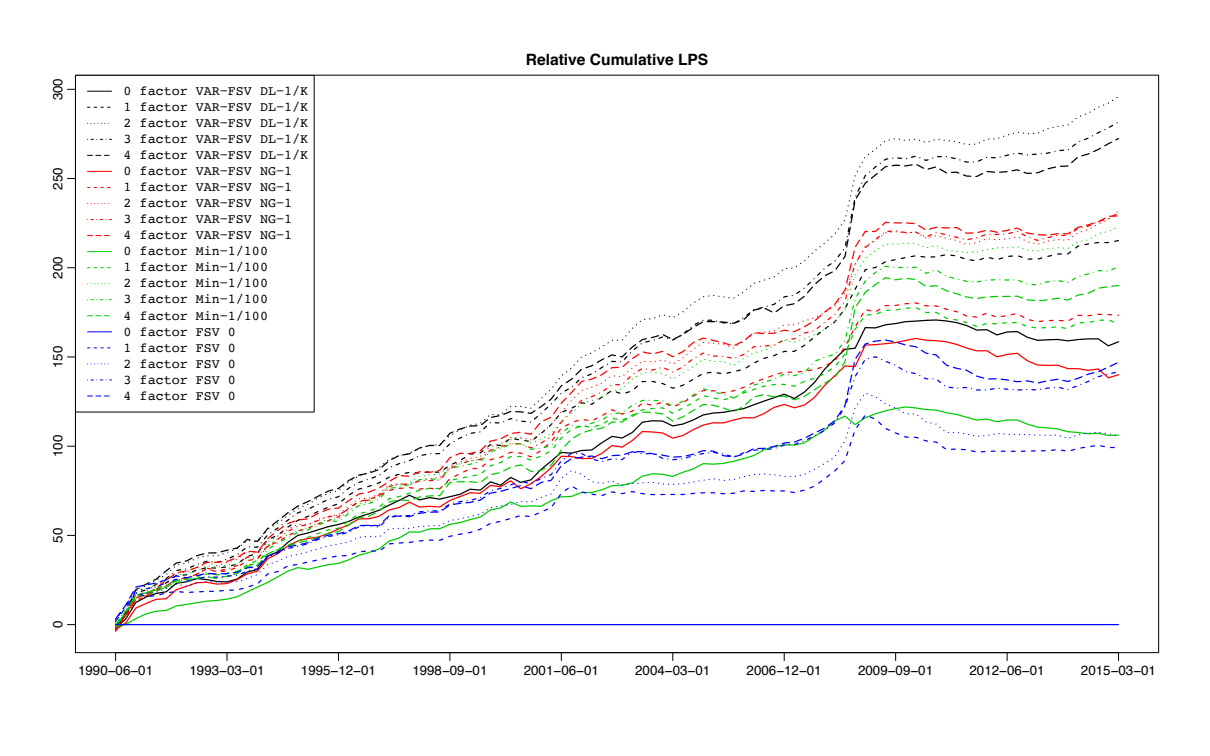
We focus on (jointly) forecasting 11 important series (cf. Koop et al., 2016)

- gross domestic product (GDPC96)
- industrial production (INPRO)
- total nonfarm payroll (PAYEMS)
- civilian unemployment rate (UNRATE)
- new privately owned housing units started (HOUST)
- consumer price index inflation (CPIAUCSL)
- producer price index for finished goods inflation (PPIFGS)
- effective federal funds rate (FEDFUNDS)
- 10-year treasury constant maturity rate (GS10)
- U.S./U.K. exchange rate (EXUSUKx)
- S&P 500 (S.P.500)

Initial estimation sample from 1959:Q3 to 1990:Q2, then sequential forecasting of one-quarter-ahead predictive densities and predictive likelihoods

11-variate cumulated log predictive likelihoods (high is good)

	0	1	2	3	4
VAR-FSV DL(1/2)	-1482	-1353	-1310	-1240	-1210
VAR-FSV DL(1/K)	-1049	-992	-912	-926	-935
VAR-FSV NG(1)	-1068	-1034	-976	-977	-978
VAR-FSV NG(0.1)	-1319	-1207	-1134	-1106	-1112
VAR-FSV Min(0.01)	-1101	-1039	-984	-1007	-1018
VAR-FSV Min(0.0001)	-1215	-1121	-1103	-1080	-1090
FSV 0	-1208	-1108	-1102	-1066	-1061
FSV 0.8	-1178	-1132	-1106	-1087	-1096
FSV 1	-1171	-1128	-1095	-1095	-1107


Relative Cumulative LPS
0 factor VAR-FSV DL-1/K
1 factor VAR-FSV DL-1/K
2 factor VAR-FSV DL-1/K
3 factor VAR-FSV DL-1/K
4 factor VAR-FSV DL-1/K
0 factor VAR-FSV NG-1
1 factor VAR-FSV NG-1
2 factor VAR-FSV NG-1
3 factor VAR-FSV NG-1
4 factor VAR-FSV NG-1
0 factor Min-1/100
1 factor Min-1/100
2 factor Min-1/100
3 factor Min-1/100
4 factor Min-1/100
0 factor FSV 0
1 factor FSV 0
2 factor FSV 0
3 factor FSV 0
4 factor FSV 0
0
50
100
150
200
250
300
1990-06-01
1993-03-01
1995-12-01
1998-09-01
2001-06-01
2004-03-01
2006-12-01
2009-09-01
2012-06-01
2015-03-01

Conlusion

- Fully Bayesian MCMC-based inference for huge dimensional VAR models with heteroskedastic variances and covariances
- Equation-by-equation estimation possible due to latent variable representation of residual covariance matrix + `factorstochvol`
- Faster algorithm (Bhattacharya et al., 2016) for sampling VAR coefficients if $T \lesssim mp$
- Continuous global-local shrinkage priors (DL prior and friends) help to cure the curse of dimensionality and provides a viable alternative to Minnesota-type priors
- Gains to be seen in simulation studies carry over to out-of-sample predictive evaluation
- Factor SV structure is crucial for macro application and improves joint density predictions markedly

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- Could be used for TVP-VARs analogously...
- ...probably by using a lower-dimensional factor structure for VAR coeffiecients as well

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